BEAM PROPAGATION METHOD APPLIED TO SINGLE AND DOUBLE STEP DISCONTINUITIES IN PLANAR OPTICAL WAVEGUIDES

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Abstract

The power transmission and loss at a discontinuity in planar optical waveguides are calculated numerically using the Beam Propagation Method (BPM) for transverse electric modes. A single step discontinuity including changes in core thickness and refractive index are considered as well as symmetric double step discontinuity. A comparison between the results obtained by this method and those obtained by three other methods shows a good agreement.

Introduction

The powers transmitted and lost at the junction between two nonidentical planar optical waveguides can be calculated by means of many well known techniques: variational[1], mode matching[2], Wiener-Hopf[3], residue calculus[4] and Green's function[5]. All of these methods are relatively complicated and usually require the solution of an infinite system of equations or the expansion of the electromagnetic field in terms of an infinite set of orthogonal functions or polynomials which are oscillatory, and this requires a great care to guarantee the stability and the convergence of the solution.

In many practical integrated optical devices, we need to fabricate two or more optical waveguides with different optogeometric properties on the same substrate. At the junction between such waveguides, there exist transmitted and reflected fields. Fortunately, in many practical cases, these nonidentical waveguides have slightly different optogeometric properties: small variations in the refractive index and the waveguide's thickness. This allows us to neglect the reflected field. In such cases the BPM[6],[7] can be used efficiently to calculate the transmitted field through a junction between two nonidentical waveguides. Of course the method can be applied to optical waveguides with two-dimensional confinement of the light power like optical fibers, but in this case a two dimensional Fourier transform is required instead of the one-dimensional transform needed for planar waveguides.
Theory

The BPM consists basically of propagating the electromagnetic field over a small distance $\Delta z$ through a homogeneous medium and then correcting for the refractive index variations during the propagation distance $\Delta z$.

Let us assume that $E_y(x,z)$ is the propagating electric field in a $y$-invariant inhomogeneous planar optical waveguide, where $\hat{y}$ is a unit vector in the $y$ direction. This field is the solution of the scalar wave equation:

$$\frac{\partial^2 E_y(x,z)}{\partial z^2} + \left( V_t + k_0 n^2(x) \right) E_y(x,z) = 0 \quad \text{(1)}$$

where $V_t$ is the transverse laplacian $\partial^2/\partial x^2$, $k_0$ is the free space wave number, and $n(x)$ is the refractive index distribution in the direction transverse to the direction of propagation $z$. The time dependence is assumed to be $\exp(i\omega t)$, where $\omega$ is the radial frequency of the electromagnetic field and $i=\sqrt{-1}$. The solution of (1) at $z=\Delta z$ may be written formally in terms of the field at $z=0$ as follows:

$$\frac{2}{\Delta z} \left( V_t + k_0 n^2(x) \right)^{1/2} E_y(x,\Delta z) = e^{2\Delta z/V_t} E_y(x,0) \quad \text{(2)}$$

The square root in (2) may be written as:

$$\left( V_t + k_0 n^2(x) \right)^{1/2} = \frac{V_t}{\left( V_t + k_0 n(x) \right)^{1/2} + k_0 n(x)} \quad \text{(3)}$$

If $n(x)$ in the first right-hand member of (3) is replaced by a certain constant value $n_s$ (typically the substrate index of refraction), an approximation to (2) takes the form:

$$\frac{2}{\Delta z} \left[ \left( \frac{V_t}{V_t + k_0 n_s} \right)^{1/2} + k_0 n(x) - n_s \right] E_y(x,\Delta z) = e^{2\Delta z/V_t} E_y(x,0) \quad \text{(4)}$$

Each squared bracket in the exponential of (4) is an operator; these two operators do not commute, but to the second order in $\Delta z$, equation (4) can be written as:

$$E_y(x,\Delta z) = P \cdot Q \cdot P \{ E_y(x,0) \} \quad \text{(5)}$$

where $P$ and $Q$ are the two operators:

$$P = e^{2\Delta z / \left( V_t + k_0 n(x) \right)^{1/2}} \quad \text{(6)}$$

$$Q = \exp\left( -i \Delta z \left( V_t + k_0 n(x) \right)^{1/2} \right) \quad \text{(6)}$$

and
\[ Q = e^{-\frac{i}{2} \Delta z (k_0[n(x) - n_0])} \]  

(7)

Operating with \( P \) on \( E_y(x,0) \) is simply a propagation of the field in a homogeneous medium (whose index is \( n_0 \)) over a distance \( \Delta z/2 \). The operator \( Q \) accounts for the variations in the index of refraction \( n(x) \). The error introduced in the solution (5) is proportional to \( (\Delta z)^2 \) and hence small propagating steps are necessary to obtain well accurate results\(^7\). The direct operation of \( P \) on \( E_y(x,0) \) involves the expansion of the exponential operator (6) in an infinite series of differential operators which are extremely difficult to treat in numerical applications. However if we consider the spatial Fourier transform of \( E_y(x,0) \), the propagation over \( \Delta z/2 \) is reduced to a simple multiplication of the transform of \( E_y(x,0) \) and the transform of the operator \( P \) which is simply\(^6\):

\[ F(P) = e^{\frac{i}{2} \Delta z (k \nabla / [(k_0 n_0^2 - k_0^2)^{1/2} + k_0 n_0])} \]  

(8)

where \( F(P) \) denotes the Fourier transform of \( P \) and \( k \) is the variable of the transform. Although in the BPM we deal with the total propagating field regardless of its modal content, we can restore easily the modal behaviour of \( E_y(x,z) \) through the eigenmode expansion\(^9\):

\[ E_y(x,z) = \sum_n t_n e_{yn}(x) e^{-\frac{i}{2} \beta_n z} + R \]  

(9)

where \( e_{yn}(x) \) is the transverse distribution of the electromagnetic field of the \( n \)-th guided mode propagating in the \( z \)-direction with a propagation constant \( \beta_n \). The radiation field in the eigenmode expansion (9) has the form of a Fourier integral which is denoted by \( R \). The constant \( t_n \) is the expansion coefficient of the \( n \)-th guided mode of the waveguiding structure. This coefficient can be calculated by taking the scalar product of (9) with the complex conjugate \( e_{yn}^*(x) \exp(i\beta_n z) \):

\[ t_n = \int_{-\infty}^{\infty} E_y(x,z) e_{yn}^*(x) \, dx \]  

(10)

\[ t_n = \int_{-\infty}^{\infty} |e_{yn}(x)|^2 \, dx \]

The power \( P_n \) carried by that mode is simply:

\[ P_n = \frac{|t_n|^2}{2} \beta_n \int_{-\infty}^{\infty} |e_{yn}(x)|^2 \, dx \]  

(11)
The propagation of the total field from \( z \) to \( z + \Delta z \) is performed through three steps: the operator \( P \) (the propagator) is applied on \( E_r(x,z) \) using the Fourier transform; the resulting field is then operated on by \( \sigma \) (the corrector) giving rise to the corrected field, finally the corrected field is operated on by \( P \) to obtain the total field at \( z + \Delta z \). This process can be continued until the desired propagation distance is reached.

**A symmetric single step discontinuity**

A single step discontinuity consisting of two butt-joined nonidentical symmetric waveguides is shown in figure 1. This case was studied previously by Marcuse\(^{19}\) using an approximate mode matching technique. Ittipiboon et al.\(^{41}\) studied the same case using residue calculus technique. Also Nishimura et al.\(^{10}\) used an approximate integral equation formulation to calculate the relative radiated power \( P_r \) (the ratio between the power lost by radiation and the power carried by the incident mode from the left of the step). Figure 1 shows the results of the BPM (solid dots) and those of the three previous techniques, where \( P_r \) is plotted as a function of \( k_0 d_1 \) when \( d_2/d_1 \) is kept constant equal to 0.5 and the wavelength is equal to 0.6328 \( \mu m \).

**An asymmetric single step**

The second case we considered was an asymmetric step which was studied rigorously by Boyd et al.\(^{11}\) using a discrete representation of the radiation field. Figure 2 shows the magnitude of the transmission coefficient \( |t_n| \) of the fundamental mode (i.e. \( n = 0 \)) as a function of \( k_0 d_1 \) when the step ratio \( d_2/d_1 = 0.5 \) at a wavelength equal to 0.6328 micron. The solid dots represent the results of the BPM.

The fairly good agreement between the results of the BPM and the other methods is due to the fact that the forward radiated power is much higher than the reflected guided and backscattered radiated power for a step ratio equal to 0.5 as pointed out by Marcuse\(^{19}\). This means that the losses are mainly due to forward scattering; and this justifies the fundamental approximation used in the BPM: the reflected field can be neglected.

**Double step**

It is well known that a dielectric optical waveguide allows, besides a finite number of guided modes, a continuum of modes. The modes within a finite range of the continuum are propagating; the rest represent energy stored locally in the neighborhood of the discontinuity (non-propagating modes). In the double step shown in figure 3, the input waveguide (to the left of the step S1) is single
mode and identical to the output waveguide (to the right of the step S2); while the intermediate guide of length L allows the first two even guided TE modes to propagate. When the fundamental mode of the input guide is incident from the left it is scattered by the step S1 in all guided modes allowed at either side of S1 as well as in the continuous spectrum of modes. After propagating away from the step S1, the guided modes and the propagating part of the continuous spectrum are again scattered by the second step S2, so that interference between the fields scattered by S1 and S2 takes place: we say that the propagating part of the continuous spectrum of modes excited at one step "sees" the adjacent step. On the other hand, the phase of the transmitted guided mode at the output waveguide depends on the length L of the intermediate guide. These interferences influence the guided transmitted field at the output waveguide and the transmission coefficient $t_{o1}$ of the fundamental mode at the output waveguide exhibits oscillations as function of L; this phenomena was pointed out by Rozzi[12], Biehlig[13] and Koshiba et.al.[14]. Figure 3 shows the results obtained by the BPM, where the period of the oscillation of $t_{o1}$ is approximately equal to 80 microns.

**Conclusion**

To the author's knowledge, the BPM applied to the problem of waveguide discontinuity is presented for the first time in this paper. The agreement between the results obtained by the BPM and those obtained by other methods is fairly good. The ability to treat a phase-sensitive discontinuity using the BPM is proved by considering a double step. We think that the BPM is one of the powerful and simple methods that can deal with optical waveguide discontinuities, and may be it will be the most convenient and the simplest method for studying the discontinuities in waveguides having arbitrary refractive index distributions.

**REFERENCES**


Figure 1 - A symmetric step discontinuity:
- Integral equation method
- Residue calculus technique
- Approximate mode matching technique
- BPM

Figure 2 - An asymmetric step discontinuity:
- Rigorous mode matching technique
- BPM
Figure 3 - A symmetric double step discontinuity: 
\( d_1 = 0.5d_2 = 3.5 \) micron and the wavelength is equal to \( 0.6328 \) micron.