Suppression of Subsynchronous Resonance Oscillations in Power Systems Using a Modified PSS

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Abstract:

In this work, a modified power system stabilizer (MPSS) is proposed for the purpose of suppressing the subsynchronous resonance (SSR) oscillations. The suggested supplementary signal fed to the proposed controller is taken to be proportional to the angular speed difference between generator rotor proximity masses. The suggested MPSS controller is applied to a simple power system consisting of a single machine with a fast acting static exciter connected to an infinite busbar through a series compensated tie line. Genetic algorithms (GAs) are applied for the determination of a set of optimal parameters for all adopted controllers.

A comparative study to the expected performance for the cases where the generator excitation is taken to be constant, or controlled by the conventional PSS, NDS against the suggested MPSS is presented when system is undergoing both small perturbations and large-scale disturbances, the results of which have confirmed the validity and efficacity of the MPSS controller.

Index Terms – Power System Stabilizer, Negative Damping Stabilizer, Genetic Algorithms, Subsynchronous Resonance.
I. INTRODUCTION

The Subsynchronous resonance phenomena in power systems was first encountered in 1970 at Mohave when an incident shaft breakdown took place. This unfortunate event recurred at the same location in 1971 [1]. The understanding of this phenomena is attributed to the torsional interactions between the shaft rotor oscillations and the electromagnetic torque produced by subsynchronous electrical currents when the complementary frequency of the electric circuit natural frequency near coincides with one of the rotor mechanical natural frequencies. The basic effect of subsynchronous resonance is the induction of a growing oscillatory torque component with the detrimental effect of loss of shaft life and can eventually result in shaft breaking due to fatigue [1,2]. Since this accident, many control strategies were proposed, and protective schemes were developed to damp out or suppress, or avoid the effects of the SSR phenomena. The adopted SSR countermeasures included the use of line filters, bypass filters and dynamic filters [3], SSR relays [3,4], control over static var compensators [5], coordination between FACTS devices [6], excitation voltage control [7], DC-link operation [8] and phase unbalance [9]. In this work a modified PSS is suggested to limit the effect of the SSR oscillations using a carefully selected speed signal derived from the actual discrepancy between relevant rotor masses as a supplementary signal fed to the voltage controller and termed a modified PSS for SSR suppression.

II. SYSTEM UNDER STUDY

The power system selected to perform is that of a single machine connected to an infinite bus bar through a transformer and a series compensated transmission line as shown in Fig. 1. The synchronous machine is represented by its equivalent dq0 full order model with two stator windings on each axis and three rotor windings. The machine rotor mechanical system comprises of a four tandem mounted masses, namely, a high-pressure turbine, an intermediate-pressure turbine, a low-pressure turbine and the generator rotor mass. The dynamics of the generator mechanical system is represented by the lumped multi-mass model. A shunt capacitor is connected at the high-tension side of the transformer to represent the line shunt capacitance and the possible effect of a SPM. The system differential equations and data are given in Appendices A and B respectively together with the nomenclature adopted presented in appendix c.

Fig.1 Configuration of the power system under study

The small perturbation form of the equations representing the system model given in Appendix A will take the form of:

$$\Delta \mathbf{X} = \mathbf{A} \Delta \mathbf{X} + \mathbf{B} \Delta \mathbf{u}$$  \hspace{1cm} (1)

The eigenvalue solution for the system when undergoing small perturbations is carried out for the case of machine with constant excitation. The results in a normalized form for the mechanical mode-shapes of the generators rotor shaft are presented in Fig. 2 together with their corresponding natural angular frequencies.
Fig. 2 Mechanical mode shapes and natural frequencies

The decrement factor of the subsynchronous electrical mode is plotted against the system’s natural angular frequency in Fig. 3 at various compensation levels ranging from 1% to 100% of line reactance. It is obvious that the system will suffer mechanical instability due to negatively damped oscillations attributed to the subsynchronous resonance phenomena within two ranges of compensation levels centered around 26% and 58%, and electrical instability due to self-excitation for compensation levels higher than 67%. In this work, attention will be focused on providing adequate control to suppress the SSR modes of instability as for the suppression of self-excitation is dealt with in two companion papers [11,12].

Fig. 3 Variation in decrement factor of subsynchronous electrical mode and mechanical modes against the corresponding natural frequency at various compensation levels
III. INTRODUCING THE CONVENTIONAL PSS AND NDS IN POWER SYSTEMS

In this section a quick preview of the effect of the controllers with supplementary signal available in the literature of power system namely PSS and NDS is presented.

a. Power System Stabilizers

Power system stabilizers (PSS) are widely used in controllers with supplementary signals derived from the rotor speed error signal $\Delta \omega$ and its output signal is added to the voltage error signal fed to the AVR. They were successful in producing positive damping to the hunting mode of oscillations in power systems [12]. The excitation system and the PSS block diagram are shown in Fig. 4 with the two phase compensators taken to be identical.

\[ \text{Fig.4 Generator with voltage control loop} \]
(a) system block diagram
(b) Conventional PSS transfer function

b. Negative Damping Stabilizers

Negative damping stabilizers (NDS) were proposed by [13] to damp self-excited oscillations in power systems with series compensated transmission lines. The supplementary signal fed to the NDS is based on to error in reactive power output of the machine and the NDS output signal is added to the AVR output signal fed to the exciter. Since the self-excited oscillations lies in the subsynchronous range of frequencies just below SSR oscillations, the NDS was expected to provide more damping over SSR mode. A low-pass filter is added to the transfer function of the NDS to block supersynchronous frequency components of the reactive power signal. The excitation system and the adopted NDS transfer functions are shown in Fig.5

\[ \text{Fig.5 Generator with voltage control loop} \]
(a) Full system block diagram
(b) Conventional NDS transfer function

IV. INTRODUCING THE PROPOSED MODIFIED PSS CONTROLLER

When subjected to a disturbance the masses of the generator shaft will respond by oscillating against each other at shaft natural frequencies forming the various mechanical mode shapes. The careful study of the mechanical modes presented in Fig. 2 results that
common to both cases of mechanical modal instability, i.e. mode 1 occurring at a compensation level of 0.26 pu and slip angular frequency of 127.9 rad/s and mode 2 at compensation level of 0.58 pu and corresponding slip angular frequency of 209.22 rad/s, the generator rotor mass and the low pressure turbine mass will maintain an antiphase oscillatory mode, thus producing large speed difference in both cases of SSR instability. The use of this speed difference is thought to provide a more significant and effective damping signal to the SSR oscillations.

Unlike the situation of conventional PSS controllers, where the supplementary signal is derived from the generator rotor angular speed deviation from the synchronous angular speed, the idea behind the suggested modified PSS controller resides in selecting its supplementary signal to be proportional to the angular speed difference between the rotor mass and the antecedent low pressure turbine mass. The excitation system and the proposed modified PSS (MPSS) transfer function are shown in Fig.6. The supplementary signal loop consists of a two stage high-pass filters to ensure that the controller will not affect the machine hunting mode and a single stage low-pass filter to block the supersynchronous frequency components of the input signal. The output of the proposed controller is fed directly to the AVR output rather than the voltage error in the normal PSS.

V. GENETIC ALGORITHMS

Genetic Algorithms (GAs) are global search algorithms based on the mechanism of natural selection evolution and genetics. They search many peaks in parallel which lowers the probability of ending at a local minimum. GAs depend in their search on an objective function to be minimized with no need for any auxiliary information about its derivative or any other mathematical model for the problem in hand. A major advantage of GAs over conventional search techniques is that the initial set of solutions, initial population, is randomly initialized as there is no need to start with an initial guess in the vicinity if the required solution. Due to their simplicity and efficiency, GAs are receiving increasing attention for the power system engineer in many fields such as power system planning, optimal capacitor placement, and tuning of controller parameters [14,15,16,17,18].

A set of possible solutions for the problem in hand, individuals, is randomly initialized. Each individual performance is assessed by an objective function and thereafter the relative fitness of each individual is calculated based on its rank among other individuals. Pairs of individuals are randomly selected for mating on a basis of the fitness value of each individual such that the more fit the individual is, the higher the opportunity it will receive to reproduce.
A crossover operator is applied on the selected pairs of individuals with a high probability value to produce offsprings. A set of the offsprings created, determined by the insertion rate, will be selected randomly according to their fitness to be inserted in the current population replacing older individuals. Since crossover operation only exchanges the existing data strings that have proved fit between individuals, a mutation operator have to be applied with a low probability to introduce new strings during evolution.

The performance of GAs can be improved by dividing the whole population into a set of subpopulation. Each subpopulation is allowed to evolve, isolated from other subpopulations, for a specified number of generations, after which the most fit individuals in each subpopulation will be forced to immigrate to another subpopulation.

**Genetic Algorithm Parameters**

The phenotypic values of the search variables are coded by gray code into binary chromosomes. The precision of binary representation is 30 digits per variable. Gains are linearly scaled while time constants follows a logarithmic scale. The adopted parameters of the genetic algorithms used are:

- Generation gap: 80%
- Insertion rate: 90%
- Crossover rate: 70%
- Mutation rate: 5%
- Migration rate: 15%
- Number of generations between migrations: 30

A stochastic universal sampling selection scheme is used to determine how many chances each individual will have to reproduce [19,20], and a double point crossover will take place between each two individuals selected for reproduction. Offsprings selected for reinsertion will replace the least fit individuals in the current population. The evolution is allowed to continue until a maximum number of generations is reached and the best-fit individual in the last generation is evaluated. The total number of subpopulations, number of individuals per subpopulation and the maximum number of generations adopted for the parameter optimization problem for each controller are given in Table I.

<table>
<thead>
<tr>
<th>Subpopulations</th>
<th>PSS</th>
<th>NDS</th>
<th>MPSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of individuals per subpopulation</td>
<td>20</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>Maximum number of generations</td>
<td>400</td>
<td>400</td>
<td>50</td>
</tr>
</tbody>
</table>

**Fitness Function**

Various forms of objective functions were used for the purpose of optimizing controller parameters. [14,15,16,18,19]. The adopted objective function $J_i$ upon which the selection of individuals to create offsprings is selected such that all eigenvalues are forced to be placed as far from the imaginary axis in the left s-plane as can be.

$$J_i = \max \{ \Re(\lambda_{ik}) \}$$

where $i=1,2,\ldots,N_{ind}$, $k=1,2,\ldots,n$ and $N_{ind}$ is the total number of individuals, $n$ is the total number of modes in the eigenvalue solution, and $\lambda_{ik}$ is the eigenvalue $k$ of the system with control parameters as given by individual $i$. 

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VI. OPTIMUM PARAMETER ALLOCATION FOR POWER SYSTEM CONTROLLERS

The power system under study was modeled and analyzed using three different types of voltage controllers namely PSS, NDS and thesuggest MPSS and for each time GAs were adopted to generate the optimum value for the controller parameters at both compensation levels where SSR instability was found to occur. Results of the optimization process for determining optimal parameters values for the various types of controllers are listed in Table II; while the fitness function improvement through generations are presented in Fig. 7.

Table II Optimal parameters for the controllers as selected by a GA

<table>
<thead>
<tr>
<th>Parameters</th>
<th>PSS</th>
<th>NDS</th>
<th>MPSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>26% compensation level</td>
<td>k = 85.503, k₁ = 88.739, k₂ = 200.0</td>
<td>T_w = 0.016157, T₁ = 1.0, T₂ = 0.42031, T₁ = 0.052373, T₂ = 3.7111, T₁ = 0.02829, T₂ = 0.046164</td>
<td>Final Fitness = -0.25265, -0.4004, -1.0</td>
</tr>
<tr>
<td>58% compensation level</td>
<td>k = 1756.9, k₁ = 417.6, k₂ = 200.0</td>
<td>T_w = 5.688, k₁ = 1.0, T₂ = 0.53882, T₁ = 0.69522, T₂ = 1.8559, T₁ = 0.021084, T₂ = 0.021658</td>
<td>Final Fitness = 1.2535, -0.27735, -1.0001</td>
</tr>
</tbody>
</table>

Fig. 7 GA optimization results for the adopted controllers
(a) PSS at 26% compensation level
(b) PSS at 58% compensation level
(c) NDS at 26% compensation level
(d) NDS at 58% compensation level
(e) MPSS at 26% compensation level
(f) MPSS at 58% compensation level
VII. SYSTEM ANALYSIS AND RESULTS

In this section the analysis of the power system suggested in section (II) is performed adopting the following control schemes: a) no voltage control, b) PSS controller, c) NDS value technique is applied to its modal matrix at the two compensation levels, t was far below the required threshold causing instability to prevail accompanied with a reduction in hunting mode the originally unstable mechanical mode but with presence controllers such as PSS, NDS and MPSS controllers.

Table III Eigenvalues for the power system with 26% series compensation

<table>
<thead>
<tr>
<th>Mode</th>
<th>Constant Excitation (No Control)</th>
<th>Excitation Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>PSS</td>
</tr>
<tr>
<td>Rotor modes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Network modes</td>
<td>-6.3409±j2602.1</td>
<td>-6.05±j2603.4</td>
</tr>
<tr>
<td></td>
<td>-8.0511±j1847.8</td>
<td>-7.388±j1845.3</td>
</tr>
<tr>
<td></td>
<td>-3.9461±j544.58</td>
<td>-3.9348±j544.61</td>
</tr>
<tr>
<td>Electrical mode</td>
<td>-4.4431±j209.12</td>
<td>-3.0213±j211.41</td>
</tr>
<tr>
<td>(m=8)</td>
<td>-1.2323±j308.84</td>
<td>-1.2934±j308.84</td>
</tr>
<tr>
<td>Mechanical mode (3)</td>
<td>1.4666±j209.07</td>
<td>-0.2525±j207.64</td>
</tr>
<tr>
<td>Mechanical mode (1)</td>
<td>-0.3223±j128.12</td>
<td>-0.2526±j127.06</td>
</tr>
<tr>
<td>Mechanical mode (0)</td>
<td>-0.6814±j10.208</td>
<td>-0.2525±j10.85</td>
</tr>
<tr>
<td>Excitation system</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-61.139</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.28845</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.25257</td>
</tr>
<tr>
<td>D- axis damper mode</td>
<td>-32.241</td>
<td>-77.986</td>
</tr>
<tr>
<td>Q-axis damper mode</td>
<td>-10.033</td>
<td>-9.7479</td>
</tr>
</tbody>
</table>

Table IV Eigenvalues for the power system with 58% series compensation

<table>
<thead>
<tr>
<th>Mode</th>
<th>Constant Excitation (No Control)</th>
<th>Excitation Control</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>PSS</td>
</tr>
<tr>
<td>Rotor modes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Network modes</td>
<td>-6.3302±j2604.5</td>
<td>-5.996±j2605.6</td>
</tr>
<tr>
<td></td>
<td>-8.0307±j1850.2</td>
<td>-7.3089±j1848.2</td>
</tr>
<tr>
<td></td>
<td>-4.0768±j627.01</td>
<td>-4.0601±j627.08</td>
</tr>
<tr>
<td>Electric mode</td>
<td>-6.209±j127.28</td>
<td>-2.8387±j130.76</td>
</tr>
<tr>
<td>(m=8)</td>
<td>-1.292±j308.84</td>
<td>-1.292±j308.84</td>
</tr>
<tr>
<td>Mechanical mode (3)</td>
<td>-0.5724±j209.06</td>
<td>-0.6245±j209.12</td>
</tr>
<tr>
<td>Mechanical mode (1)</td>
<td>5.1014±j127.29</td>
<td>1.2535±j125.49</td>
</tr>
<tr>
<td>Mechanical mode (0)</td>
<td>-1.0346±j11.054</td>
<td>1.2535±j10.28</td>
</tr>
<tr>
<td>Excitation system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>modes</td>
<td>-0.2909</td>
<td>-27.514±j42.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.5515±j10.2423</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.15624</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D- axis damper mode</td>
<td>-32.717</td>
<td>-85.269</td>
</tr>
</tbody>
</table>

For the case of 26% compensation level, the conventional PSS brought stability to the originally unstable mechanical mode but with marginal value of stability index and was accompanied with a reduction in hunting mode damping down to 37% of its uncontrolled value, thus causing the system to oscillate at the hunting frequency for a period of over 20 seconds. On the other hand, for level of compensation of 58% the damping effect of the PSS was far below the required threshold causing instability to prevail.
The NDS controller with optimized parameters did exhibit a better performance when compared to the conventional PSS as it provided the required damping over SSR oscillations for both cases of instability, but here also the damping effect to the hunting mode was slightly reduced. Finally, the performance of the suggested MPSS is found to be more effective in stabilizing the system as it not only suppressed the SSR instability but did increase the damping over all other mechanical modes although some reduction in the damping of the subsynchronous electrical mode of oscillation was observed.

The time solution to the system equation representing the power system under study in its linearized form is performed for a relatively small perturbation which is expressed in the form of a sudden reduction in mechanical input torque of 10% of its steady state value and was sustained for a duration of 100msec after which the input torque was restored to its initial value. Using the MPSS controller parameters previously derived at 0.58 pu compensation level, the eigenvalues for the system were extracted for all practical compensation levels, the results obtained are depicted in fig.8.

The time response for the generator rotor angle for the various types of controllers at the two levels of compensation is depicted in Fig.9 and Fig.10. The correlation between the outcome of the state space solution and time solution confirms the validity of the results produced, where the cases of compensation levels of the order of 0.58 pu were unstable at both the no control and the PSS controller configurations while the system did conserve its stable state for all other cases.

**b. System subjected to a large scale disturbance**

The time solution for the prototype power system previously adopted is repeated again for all cases mentioned above but this time when system was subjected to a large-scale disturbance expressed in the form of a symmetrical three-phase short circuit applied at the high tension busbar of the power transformer lasting for a duration of 80 ms and thereafter fault is removed hence restoring the system to its initial status. Here the controller parameters were selected as previously derived in section (VI). The rotor angle time variation at the compensation levels known to cause SSR instability is presented in Fig.11 and Fig.12.

From the results portrayed it can be concluded that for the compensation level of 0.26pu the system retained its stability for all types of selected controllers although for the case of conventional PSS controller the damping to the oscillatory torque is exceedingly low, and for the case of NDS controller the damping to subsynchronous oscillations superimposed on the variation of the rotor angle is noticeably low. As for the compensation level of 0.58 pu only the MPSS controller did successfully retain system stability. This confirms the effectiveness of the modified PSS controller over all SSR modes of instability.
Fig. 9 Rotor angle time variations (in degrees) for the system with 26% series compensation level for a 100ms 10% reduction in mechanical torque
(a) with constant excitation
(b) with PSS
(c) with NDS
(d) with MPSS

Fig. 10 Rotor angle time variations (in degrees) for the system with 58% series compensation level for a 100ms 10% reduction in mechanical torque
(a) with constant excitation
(b) with PSS
(c) with NDS
(d) with MPSS
Fig. 11 Rotor angle time variations (in degrees) for the system with 26% series compensation level for an 80ms three phase short circuit
(a) with constant excitation
(b) with PSS
(c) with NDS
(d) with MPSS

Fig. 12 Rotor angle time variations (in degrees) for the system with 58% series compensation level for an 80ms three phase short circuit
(a) with constant excitation
(b) with PSS
(c) with NDS
(d) with MPSS
VII CONCLUSION

In this work a modified version of power system stabilizer dedicated to suppress the detrimental effect of the electromechanical interaction causing oscillations stemming from the SSR phenomena is presented.

The application of the suggested MPSS to a simple single machine infinite bus bar power system is presented. Use is made of the genetic algorithms to produce optimized parameters for selected types of controller namely the conventional PSS, NDS and the suggested MPSS. The results of the comparative analysis to these controllers under both linear and non linear system representation are included. The conventional PSS and NDS with optimized parameters on one hand did marginally maintain system stability when power system was undergoing minor disturbances but have failed to maintain stability when system was exposed to a large disturbance at compensation level of 0.58 pu. The MPSS with optimized parameters on the other hand did produce total damping to the SSR oscillations under both small oscillation and large scale disturbances thus maintaining both the power system dynamic and transient stabilities. The validity of the suggested modified version of PSS controller in suppressing SSR instability was confirmed with the added benefit of increasing the damping over all other mechanical modes.

VIII. REFERENCES

1. M.C.Hall, and D.A.Hodges, “Experience with 500kV Subsynchronous Resonance and Resulting Turbine Generator Shaft Damage at Mohave Generating Station”, IEEE Winter Meeting and Tesla Symposium, pp22-29


IX. APPENDICES

Appendix A

Mathematical description for a single machine connected to an infinite power grid through a compensated tie line as shown in Fig. 1 is presented as follows.

a. Mathematical Model of The Electrical System Dynamics

\[ \frac{p}{\omega_s} \left( -x_{qf}i_d + x_{qf}i_q + x_{pf}i_d \right) = r_{i_d} + \frac{\alpha_{gen}}{\omega_s} \left( -x_{qf}i_q + x_{pf}i_q \right) + v_{id} \]

\[ \frac{p}{\omega_s} \left( -x_{qf}i_q + x_{pf}i_q \right) = r_{i_q} - \frac{\alpha_{gen}}{\omega_s} \left( -x_{qf}i_d + x_{pf}i_d + x_{qf}i_q \right) + v_{iq} \]

\[ \frac{p}{\omega_s} \left( -x_{qf}i_d + x_{pf}i_d + x_{qf}i_q \right) = -r_{i_d}i_d + v_{fd} \]

\[ \frac{p}{\omega_s} \left( -x_{qf}i_q + x_{pf}i_q \right) = -r_{i_q}i_q \]

\[ \frac{p}{\omega_s} v_{id} = x_{ip}i_d - i_{ld} + \frac{\alpha_{gen}}{\omega_s} v_{iq} \]

\[ \frac{p}{\omega_s} v_{iq} = x_{ip}i_q - i_{iq} - \frac{\alpha_{gen}}{\omega_s} v_{id} \]
\[
\frac{p}{\omega_s}\left( -x_{pl}i_{dq} \right) = r_{pl}i_{dq} - \frac{\alpha_{gen}}{\omega_s} x_{pl}i_{dq} + v_{id} - v_{cd}
\]
\[
\frac{p}{\omega_s}\left( x_{pl}i_{dq} \right) = r_{pl}i_{dq} + \frac{\alpha_{gen}}{\omega_s} x_{pl}i_{dq} + v_{id} - v_{cq}
\]
\[
\frac{p}{\omega_s}\left( x_{cd} v_{dq} \right) = x_{cd} i_{dq} + \frac{\alpha_{gen}}{\omega_s} \left( v_{cq} - v_{dq} \right)
\]
\[
\frac{p}{\omega_s}\left( v_{cd} - v_{dq} \right) = x_{cd} i_{dq} - \frac{\alpha_{gen}}{\omega_s} \left( v_{cd} - v_{dq} \right)
\]
where \( x_{di} = x_d + x_{tr}, x_{iq} = x_q + x_{tr}, r_i = r_a + r_{tr} \)

b. Mathematical Model of The Mechanical System Dynamics
\[
\frac{2H_{HP}}{\omega_s} p\omega_{HP} = -(D_{HP} + D_{HI}) \frac{\alpha_{HP}}{\omega_s} + \frac{\alpha_{gen}}{\omega_s} k_{HI} \delta_{HP} + k_{HI} \delta_{IP} + T_{IP}
\]
\[
p\delta_{IP} = \omega_{IP} - \omega_t
\]
\[
\frac{2H_{LP}}{\omega_s} p\omega_{LP} = -(D_{HL} + D_{IL}) \frac{\alpha_{LP}}{\omega_s} + \frac{\alpha_{gen}}{\omega_s} k_{IL} \delta_{LP} - \frac{\alpha_{gen}}{\omega_s} k_{IL} \delta_{LP} + T_{LP}
\]
\[
p\delta_{LP} = \omega_{LP} - \omega_t
\]
\[
\frac{2H_{gen}}{\omega_s} p\omega_{gen} = -(D_{Lg} + D_{gl}) \frac{\alpha_{gen}}{\omega_s} + \frac{\alpha_{gen}}{\omega_s} k_{Lg} \delta_{LP} - \frac{\alpha_{gen}}{\omega_s} k_{Lg} \delta_{LP} - T_{em}
\]
\[
p\delta_{LP} = \omega_{LP} - \omega_t
\]
\[
\text{as } T_{em} = \left( x_{d'd'} + x_{a'd'} - x_{a'd'} \right) + \left( x_{a'd'} + x_{a'd'} \right)
\]

Appendix B

Machine Data
\[
x_d = 1.5 \quad x_{ad} = 1.31 \quad x_{jd} = 1.42 \quad x_{kd} = 1.4
\]
\[
x_q = 1.49 \quad x_{aq} = 1.29 \quad x_{kq} = 1.34
\]
\[
r_a = 0.0015 \quad r_{jd} = 0.0063 \quad r_{ad} = 0.0153 \quad r_{kq} = 0.00207
\]
\[
D_{HP} = 0.4 \quad D_{IP} = 0.3 \quad D_{LP} = 0.3 \quad D_{gen} = 0.0
\]
\[
D_{HI} = 0.4 \quad D_{IL} = 0.4 \quad D_{Lg} = 0.4
\]
\[
k_{HI} = 47.0 \quad k_{IL} = 61.0 \quad k_{Lg} = 74.0
\]
\[
H_{HP} = 0.1883 \quad H_{IP} = 0.3351 \quad H_{LP} = 0.7562 \quad H_{gen} = 1.0
\]

Transformer Data
\[
x_{tr} = 0.135 \quad r_{tr} = 0.003
\]

Transmission Line Data
\[
x_{fl} = 1.205 \quad r_{fl} = 0.025 \quad x_{cp} = 10
\]

Exciter Data
\[
k_e = 400.0 \quad T_e = 0.01
\]

System Operating Conditions
\[
V^* = 1.0 \quad P = 0.8 \quad Q = 0.6 \quad \omega_i = 377 \text{ rad/s}
\]
Appendix C

Nomenclature

- \( x_d \): denotes the direct axis inductive reactance in pu
- \( x_q \): denotes the quadrature axis inductive reactance in pu
- \( x_{ad} \): denotes the direct axis mutual inductive reactance in pu
- \( x_{aq} \): denotes the quadrature axis mutual inductive reactance in pu
- \( x_{fd} \): denotes the field windings inductive reactance in pu
- \( x_{fd} \): denotes the direct axis damper windings inductive reactance in pu
- \( x_{fdq} \): denotes the quadrature axis damper windings inductive reactance in pu
- \( r_a \): denotes the armature resistance in pu
- \( r_f \): denotes the field windings resistance in pu
- \( r_{fd} \): denotes the direct axis damper windings resistance in pu
- \( r_{fdq} \): denotes the quadrature axis damper windings resistance in pu
- \( x_{tr} \): denotes the transformer leakage inductive reactance in pu
- \( r_{tr} \): denotes the transformer series resistance in pu
- \( x_{s} \): denotes the transmission line series inductive reactance in pu
- \( r_{s} \): denotes the transmission line series resistance in pu
- \( x_{p} \): denotes the reactance of the shunt capacitor at the high tension busbar in pu
- \( x_{s} \): denotes the reactance of the series compensation in pu
- \( H_{HP,IP,LP} \): denotes the inertia constant of the high, intermediate and low pressure turbine in sec
- \( H_{gen} \): denotes the inertia constant of the generator rotor in sec
- \( k_{HI} \): denotes the stiffness constant of the shaft section HP/IP in pu torque/rad
- \( k_{IL} \): denotes the stiffness constant of the shaft section IP/LP in pu torque/rad
- \( k_{LG} \): denotes the stiffness constant of the shaft section LP/G in pu torque/rad
- \( D_{HP} \): denotes the damping coefficient of the high pressure turbine in pu torque/(rad sec\(^{-1}\))
- \( D_{IP} \): denotes the damping coefficient of the intermediate pressure turbine in pu torque/(rad sec\(^{-1}\))
- \( D_{LP} \): denotes the damping coefficient of the low pressure turbine in pu torque/(rad sec\(^{-1}\))
- \( D_{gen} \): denotes the damping coefficient of the generator rotor in pu torque/(rad sec\(^{-1}\))
- \( D_{HI} \): denotes the damping coefficient of the shaft section HP/IP in pu torque/(rad sec\(^{-1}\))
- \( D_{IL} \): denotes the damping coefficient of the shaft section IP/LP in pu torque/(rad sec\(^{-1}\))
- \( D_{Lm} \): denotes the damping coefficient of the shaft section LP/G in pu torque/(rad sec\(^{-1}\))
- \( T_{HP} \): denotes the mechanical torque of the high pressure turbine in pu
- \( T_{IP} \): denotes the mechanical torque of the intermediate pressure turbine in pu
- \( T_{LP} \): denotes the mechanical torque of the low pressure turbine in pu
- \( T_{em} \): denotes the generator electromagnetic torque in pu
- \( k_e \): denotes the gain of the excitation system
- \( T_{e} \): denotes the time constant of the excitation system
- \( \omega_{HI,IP,LP} \): denotes the angular frequency of the high, intermediate and low pressure turbine mass in rad/sec
- \( \omega_{gen} \): denotes the angular frequency of the generator rotor mass in rad/sec
- \( \omega_k \): denotes the synchronous angular frequency in rad/s
- \( \delta_{HP,IP,LP} \): denotes the angle of the high, intermediate and low pressure turbine in radians
- \( \delta_{gen} \): denotes the generator rotor angle in radians
- \( i_d \): denotes the direct axis component of the armature current in pu
- \( i_q \): denotes the quadrature axis component of the armature current in pu
- \( i_{td} \): denotes the field current in pu
- \( i_{td} \): denotes the current in the direct axis damper windings in pu
- \( i_{tdq} \): denotes the current in the quadrature axis damper windings in pu
- \( i_{td} \): denotes the direct axis component of the transmission line current in pu
- \( i_{tdq} \): denotes the quadrature axis component of the transmission line current in pu
- \( v_{td} \): denotes the direct axis component of the voltage of the transformer high tension busbar in pu
- \( v_{tdq} \): denotes the quadrature axis component of the voltage of the transformer high tension busbar in pu
- \( v_{td} \): denotes the direct axis component of the voltage of the series capacitor busbar in pu
- \( v_{tdq} \): denotes the quadrature axis component of the voltage of the series capacitor busbar in pu
- \( v_d \): denotes the direct axis component of the infinite busbar voltage in pu
- \( v_q \): denotes the quadrature axis component of the infinite busbar voltage in pu
- \( P \): denotes the steady state active power flow at the infinite busbar
- \( Q \): denotes the steady state reactive power flow at the infinite busbar