Effect of System States and Parametric Sensitivity Analyses on Power System Modal Interactions

M.Soliman* A.A.Ishak**

Abstract

In this paper, the parametric sensitivities of a structure preserving a power system model are derived in the interest of effective model reduction. This parametric sensitivity analysis defines the sensitivity of a steady state point against the variation of a particular system parameter. The parametric sensitivities are derived here by studying the effects changes in a given parameter over the system natural modes of the variations on the small signal stability and the results obtained were confirmed. Also the correlations between different inherent modes resulting from small perturbation stability have been confirmed.

Keywords: Participation Factor, Sensitivity analysis, modal interaction, power system stability, small perturbation analysis.

1. Introduction

The state of stability of power system is not only defined by the rate of decaying oscillations or its positive damping but also by the stability of its parameters. It is also important to identify the state variable and parameters that contribute to developing a particular type of the effect of instability. In this work a comprehensive dynamic model is developed to study the system parameter over the resulting system stability using a dq0 model. The electromechanical oscillations and their damping, as well as dynamic voltage stability between a remotely located synchronous machine and the remaining of a power system are studied. A single-machine infinite-bus system case that investigates only local oscillations' sensitivities to parametric variation is introduced. The effect of parameters variation in machine parameters such as R_a, R_f, R_{kd}, R_{kq}, H, tie line parameters such as R_e, X_e and loading conditions such as P, Q, over power system natural modes is analyzed.

*Assistant Lecturer at Electrical Engineering Department, Faculty of Engineering, Benha University
**Associate Professor at Electrical Engineering Department, Faculty of Engineering, Benha University
Due to the possible large number of modal interactions it is often necessary to construct a reduced order model for dynamic stability studies by retaining only modes of interest while preserving the consistency of the analysis. The appropriate identification of the state variables significantly participating in a given mode becomes effective in defining the end form of the reduced order model. This requires a tool for identifying the state variables that have a significant participation in a selected mode. It is natural to suggest that the significant state variables for an eigenvalue \( \lambda_i \) are those that correspond to large entries in the corresponding eigenvector \( \nu_i \). Verhese et al. [1] have suggested a related measure of a state variable participation factors (PF). Participation factor analysis assists in the identification of how much each dynamic state variable affects a given mode or eigenvalue.

2. Preliminary

In power system analysis, mathematical equations that represent the power system under study in the dq0 model are generally described by seven nonlinear differential equation for the case where only one damper circuit on each axis is considered as shown in appendix [I]. This set of nonlinear differential equations can be linearized around a quiescent operating point on a basis of small perturbation. The mathematical description thus obtain yield a linearized system of differential equation with constant coefficient which will take the form:

\[
\dot{X} = AX + BU
\]

The Participation factor is a sensitivity measure of an eigenvalue to a diagonal entry of the linearized system matrix, and can be defined as

\[
PF_{ki} = \frac{\partial \lambda_i}{\partial a_{kk}}
\]

Where, \( PF_{in} \) is the participation factor relating the \( k^{th} \) state variable with the \( i^{th} \) eigenvalue, \( a_{kk} \) is the \( k^{th} \) entry of the system matrix \( A \).

It is well known that if \( A \) has all its eigenvalues \( \lambda_i \ (i=1,2,\ldots,m) \), then it will have \( m \) corresponding linearly independent \( m \times 1 \) eigenvectors \( \nu_i \ (i=1,2,\ldots,m) \) satisfying the relation

\[
A \nu_i = \lambda_i \nu_i \ (i = 1,2,\ldots,m)
\]
Here \( v_i \) is called the right eigenvector associated with \( \lambda_i \). There also exists a vector \( \omega'_i \) satisfying the relation,
\[
\omega'_i A = \omega'_i \lambda_i, \quad (i = 1,2, \ldots, m)
\]
Where \( t \) denotes matrix transposition and this vector is the left eigenvector.

Hence \( PF \) can be defined as
\[
PF_{ki} = \frac{\omega_{ki} v_{ki}}{\omega_i' v_i} \quad (i = 1,2, \ldots, m)
\]

Where \( \omega_{ki} \) and \( v_{ki} \) are \( k^{th} \) entries in the right and left eigenvectors associated with the \( i^{th} \) eigenvalue \( \lambda_i \). Equivalence between two definitions of the participation factor can be derived considering the system

\[
[A - \lambda_i I] v_i = 0.0
\]
\[
\omega'_i [A - \lambda_i I] = 0.0
\]

To examine the sensitivity of the eigenvalue \( \lambda_i \) of the diagonal element of the system modal matrix \( A \); The perturbed equations will read;

\[
(A + \Delta A)(v_i + \Delta v_i) = (\lambda_i + \Delta \lambda_i)(v_i + \Delta v_i)
\]
\[
[A v_i] + [\Delta A v_i + \Delta v_i A] + [\Delta v_i \Delta A] = [\lambda_i v_i] + [\Delta \lambda_i v_i + \Delta v_i \lambda_i] + [\Delta v_i \Delta \lambda_i]
\]

After appropriate mathematical manipulations, will yield
\[
\omega'_i \Delta A v_i = \omega'_i \Delta \lambda_i v_i
\]

Assuming that the \( k^{th} \) diagonal element of matrix \( A \) is perturbed so that \( \Delta A = \Delta a_{kk} \), hence

\[
A = \begin{bmatrix}
  a_{11} & a_{12} & \cdots & a_{1m} \\
  a_{21} & a_{22} & \cdots & a_{2m} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{m1} & \cdots & a_{kk} & \cdots \\
\end{bmatrix}
\]
\[
\Delta A = \begin{bmatrix}
  0 & 0 & \cdots & 0 \\
  0 & 0 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & \cdots & \Delta a_{kk} & \cdots \\
\end{bmatrix}
\]
Now the sensitivity of the eigenvalue $\lambda_i$ with respect to diagonal elements of the matrix $A$ is related to the $PF$ as follows:

$$\frac{\Delta \lambda_i}{\Delta a_{kk}} = \frac{\omega_i}{\omega_i} v_i = PF_{ki}$$

### 3. Study Case

The above mentioned methodology is applied to a single-machine infinite-bus system whose detailed power system model is described in appendix (I).

This formulation includes both the generator electrical and mechanical models. The analysis is performed after the machine is undergoing a small perturbation; the state space representation for system model in small perturbed form is given in appendix (II). The perturbed system equations in matrix form is given as:

$$E \Delta X^* = F \Delta X$$

$$\Delta X^* = E^{-1} F \Delta X$$

$$= Asys \Delta X$$

Where

$$Asys = E^{-1} F$$

and

$E$ denotes the matrix of coefficient of the derivative of the perturbed state variables

$F$ denotes the matrix of coefficient of the perturbation state variables

As no feedback control action is considered, it goes without saying that the matrix $\Delta u$ of the perturbed input state variables becomes a null matrix.
4. Results

1- Eigen vector P.F

The eigenvalues and eigenvectors of the perturbed system matrix $A_{sys}$ are extracted. The different modes obtained are listed below in table I.

<table>
<thead>
<tr>
<th>Eigenvalues $(\lambda_i)$</th>
<th>Stator Modes</th>
<th>Rotor Modes</th>
<th>Electrical Modes</th>
<th>Mechanical Modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_d$</td>
<td>-0.2404</td>
<td>0.2991 - 0.0037i</td>
<td>-0.35166</td>
<td>-0.6559</td>
</tr>
<tr>
<td>$i_q$</td>
<td>0.77895</td>
<td>0.0009 + 0.5259i</td>
<td>0.043367</td>
<td>0.5513</td>
</tr>
<tr>
<td>$i_f$</td>
<td>-0.3206 - 0.0215i</td>
<td>0.037814</td>
<td>-0.2404</td>
<td>0.040568</td>
</tr>
<tr>
<td>$i_{kd}$</td>
<td>-0.030526</td>
<td>0.2792 + 0.0197i</td>
<td>-0.75081</td>
<td>-0.034851+0.0008586i</td>
</tr>
<tr>
<td>$i_{kq}$</td>
<td>-0.0032287</td>
<td>-0.0087 + 0.5063i</td>
<td>0.00026214</td>
<td>0.5513</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>0.2323 + 0.0354i</td>
<td>0.073243</td>
<td>-0.06472</td>
<td>0.79617</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0001 + 0.0006i</td>
<td>0.30467</td>
<td>0.0020189</td>
<td>0.082142</td>
</tr>
</tbody>
</table>

The participation factor technique is there after applied to the eigen vectors. The results of which are given in table II and are named after the predominant dynamic variable.

<table>
<thead>
<tr>
<th>State Variables</th>
<th>Stator &amp; Network mode</th>
<th>Rotamors</th>
<th>Field mode</th>
<th>D-damper</th>
<th>Q-damper</th>
<th>Hunting</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_d$</td>
<td>0.34793</td>
<td>0.399150</td>
<td>0.02908</td>
<td>0.11366</td>
<td>0.066903</td>
<td></td>
</tr>
<tr>
<td>$i_q$</td>
<td>0.34561</td>
<td>0.011011</td>
<td>0.00000</td>
<td>0.21351</td>
<td>0.050971</td>
<td></td>
</tr>
<tr>
<td>$i_f$</td>
<td>0.06999</td>
<td>0.481500</td>
<td>0.18479</td>
<td>0.00900</td>
<td>0.062181</td>
<td></td>
</tr>
<tr>
<td>$i_{kd}$</td>
<td>0.08459</td>
<td>0.075657</td>
<td>0.78555</td>
<td>0.00568</td>
<td>0.010270</td>
<td></td>
</tr>
<tr>
<td>$i_{kq}$</td>
<td>0.15172</td>
<td>0.025197</td>
<td>0.00000</td>
<td>0.36963</td>
<td>0.044229</td>
<td></td>
</tr>
<tr>
<td>$\Omega$</td>
<td>0.00000</td>
<td>0.007025</td>
<td>0.00027</td>
<td>0.27184</td>
<td>0.382730</td>
<td></td>
</tr>
<tr>
<td>$\Delta$</td>
<td>0.00014</td>
<td>0.000462</td>
<td>0.00030</td>
<td>0.01670</td>
<td>0.382720</td>
<td></td>
</tr>
</tbody>
</table>

2- Effect of Parameter Variation

The effect of parameter variation on each mode is applied to the set of modes to work out their sensitivity of each of the predominant. This analysis will be performed in three major steps:  
a. Study of the variations in generator transformer parameters.  
b. Study of the variations in transmission system parameters.  
c. Study of the variations in initial loading condition.
a) **Effect of Variation in Generator/Transformer Parameters**

To assess the effect of the parametric variation on the inherent modes, eigenvalue program is to be executed several times, with a certain step of variations of parameters under consideration. The selected machine parameters under consideration are armature $R_a$ or transformer resistance $R_{tr}$, field resistance $R_f$, direct and quadrature axis damper resistance $R_{kd}$, $R_{kq}$ respectively, rotor inertia (2H). The system inherent modes are represented by recording the variation in both damping factors, and the angular frequency of oscillations. The most sensitive modes are to be shown.

b) **Effect of Variation in Loading Condition**

a. **Effect of Variation in Circuit External impedance**

Here the eigenvalue program is to be repeated several times, keeping the constant variation in both external resistance and reactance of the tie line. The most sensitive modes are to be shown.
Fig. 2- Effect of variation in $R_a$ and $X_{TL}/R_{TL}$ ratio on the *stator* mode.

Fig. 3- Effect of variation in $R_{fd}$, $R_{kd}$, $X_{TL}/R_{TL}$ ratio and $\cos\phi$ on the *field* mode.
Fig. 4- Effect of the variation in $R_{fd}$ and $R_{kd}$ on the D-damper mode.

Fig. 5- Effect of the variation in $R_{fd}$, $H$, $X_{TL}/R_{TL}$ ratio and $\cos\phi$ on the Q-damper mode.
Fig. 6 - Effect of the variation in $R_{fd}, R_{kd}, R_{kq}, H, X_{TL}/R_{TL}$ ratio and $\cos \phi$ on the Hunting mode.
5. Conclusion

6. References
Appendix I

System Data from reference [21] on a basis of 500 KV, 10 GW

Generator Data: $X_d = 1.51$, $X_q = 1.31$, $X_t = 0.2$, $X_{aq} = 1.49$, $X_{ad} = 1.29$, $X_{kq} = 1.4$, $X_{kd} = 1.34$.

$P_r = 0.0015$, $R_s = 0.00063$, $X_{ad} = 0.0207$, $2H = 5.24$ sec, $D = 0$

Transformer Data: $R_r = 0.003–0.0045$, $X_{pr} = 0.135$

Transmission Line Data $R_q = 0.02–0.027$, $X_q = 0.905$

Receiving System $R_s = 0.005$, $X_s = 0.3$

Initial loading condition at infinite bus bar, $V^o = 1.0$, $P_x = 0.8$, $Q_x = 0.6$, $\omega = 377.0$ rad/sec $P_e = 0.81$

All values mentioned above are taken in p.u on a basis of machine rating unless otherwise stated

Appendix II

Mathematical Model for a Single Machine – Infinite Bus Bar System

The mathematical description of the transient model for single machine infinite bus bar system shown in fig.1 is given below with state space vector $X' = [i_d, i_q, i_{qf}, i_{kd}, i_{kq}, \omega_r, \delta]$: 

\[
\frac{P}{\omega_0}(-X_{d}i_d + X_{ad}i_f + X_{ad}i_{kd}) = R_i i_d + \omega_r(-X_{d}i_d + X_{ad}i_{kd}) + V^{o} \sin \delta
\]

\[
\frac{P}{\omega_0}(-X_{q}i_q + X_{ad}i_{kq}) = R_i i_q - \omega_r(-X_{d}i_d + X_{ad}i_{kd}) + V^{o} \cos \delta
\]

\[
\frac{P}{\omega_0}(-X_{ad}i_d + X_{f}i_f + X_{ad}i_{kd}) = -R_f i_f + V_f
\]

\[
\frac{P}{\omega_0}(-X_{aq}i_q + X_{kq}i_{kq}) = -R_{kq} i_{kq}
\]

\[
\frac{2H}{\omega_0} \omega_r = T_m - T_{em} - \frac{D}{\omega_0} \omega_r, \text{ where } T_{em} = \psi_d i_d - \psi_q i_q
\]

\[
P \omega_r = \omega_r
\]

Where $X_{d} = X_d + X_t + X_{ad} + X_{qf}$, $X_{q} = X_q + X_{ad} + X_{qf}$, $R_i = R_q + R_{tr} + R_{td}$, $X = X_d - X_q$.

The steady state machine equation is derived from the system of equations given above by making the following substitutions:

(a) The operator $P = \frac{d}{dt} = 0$, (b) The per unit slip ratio $\frac{\omega_r}{\omega_0} = 1$ and (c) the steady state damper current $i_{kdo} = i_{kqo} = 0$.

$V_{Gd} = X_q i_q - R_d i_q$ and $E_f = V_{Gd} + R_{dl} i_d + X_{ad} i_d = \frac{X_{ad}}{R_f} V_f$
Loci of Stator Modes Traced by Armature Resistance (Ra)

Damping Factor (1/sec)

Armature Resistance Ra (P.U)

Oscillatory Frequency (rad/sec)

Loci of Field Modes Traced by Armature Resistance (Ra)

Damping Factor (1/sec)

Armature Resistance
Loci of Direct Damper Modes Traced by Armature Resistance (Ra)

Armature Resistance Ra (P.U)

Loci of Quadrature Damper Modes Traced by Armature Resistance (Ra)

Armature Resistance Ra (P.U)

$R_f$: 
Loci of Hunting Modes Traced by Field Resistance ($R_f$)

Loci of Direct Damper Modes Traced by Field Resistance ($R_f$)
Loci of Quadrature Damper Modes Traced by Field Resistance (Ra)

Field Resistance RF (P.U)
Damping Factor (1/sec)
Hunting Modes Traced by Q-Damper Resistance ($R_{q\omega}$)

Damping and Frequency of hunting (p.u.)

Q-Damper Resistance (p.u.)
Hunting Modes Traced by Field Resistance (RF)

Loci of Field Modes Traced by Field Resistance (RF)
**Stator Modes Traced by Tie Line Reactance (XL)**

![Graph showing the relationship between Damping frequency of Stator Modes (p.u) and Tie Line Reactance XL (p.u).]

**Loci of Field Modes Traced by Tie Line Reactance (XL)**

![Graph showing the relationship between Damping of Field Mode (p.u) and Tie Line Reactance XL (p.u).]
Hunting Modes Traced by Tie Line Reactance (XL)

Loci of Q-Damper Modes Traced by Tie Line Reactance (XL)