Three-phase Circuits

Unbalanced 3-phase systems

Power in 3-phase system

UNBALANCED DELTA-CONNECTED LOAD

The line currents will not be equal nor will they have a 120° phase difference as was the case with balanced loads.

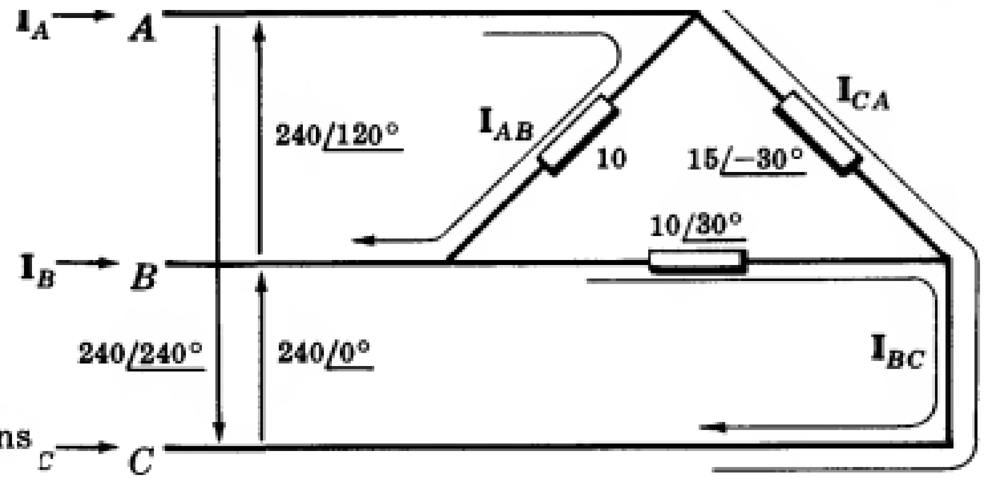
Example 4.

A three-phase, three-wire, 240 volt, *ABC* system has a delta-connected load with $\mathbf{Z}_{AB} = 10\underline{/0^\circ}$, $\mathbf{Z}_{BC} = 10\underline{/30^\circ}$ and $\mathbf{Z}_{CA} = 15\underline{/-30^\circ}$. Obtain the three line currents and draw the phasor diagram.

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_{AB}} = \frac{240\underline{/120^\circ}}{10\underline{/0^\circ}} = 24\underline{/120^\circ}$$

$$\mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_{BC}} = 24\underline{/-30^\circ}$$

$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_{CA}} = 16\underline{/270^\circ}$$



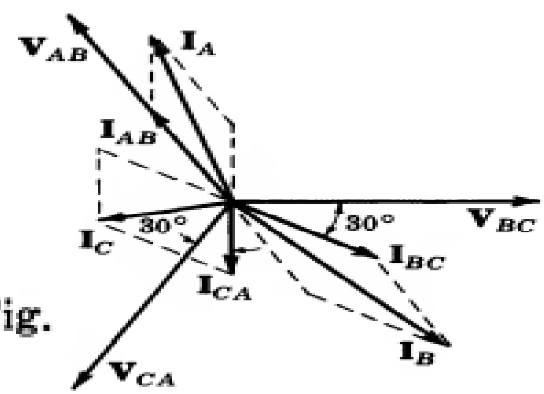
Apply Kirchhoff's current law to the junctions

$$\mathbf{I}_A = \mathbf{I}_{AB} + \mathbf{I}_{AC} = 24\underline{/120^\circ} - 16\underline{/270^\circ} = 38.7\underline{/108.1^\circ}$$

$$\mathbf{I}_B = \mathbf{I}_{BA} + \mathbf{I}_{BC} = -24\underline{/120^\circ} + 24\underline{/-30^\circ} = 46.4\underline{/-45^\circ}$$

$$\mathbf{I}_C = \mathbf{I}_{CA} + \mathbf{I}_{CB} = 16\underline{/270^\circ} - 24\underline{/-30^\circ} = 21.2\underline{/190.9^\circ}$$

The corresponding phasor diagram is shown in Fig.

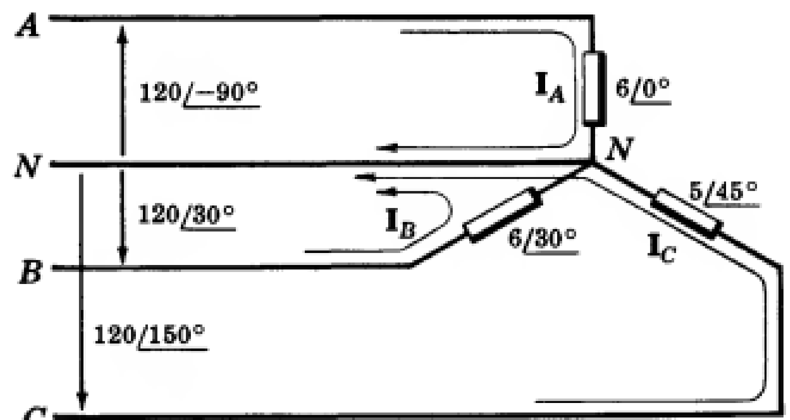


UNBALANCED FOUR-WIRE, WYE-CONNECTED LOAD

- On a four-wire system the neutral conductor will carry a current when the load is unbalanced
- The voltage across each of the load impedances remains fixed with the same magnitude as the line to neutral voltage.
- The line currents are unequal and do not have a 120° phase difference.

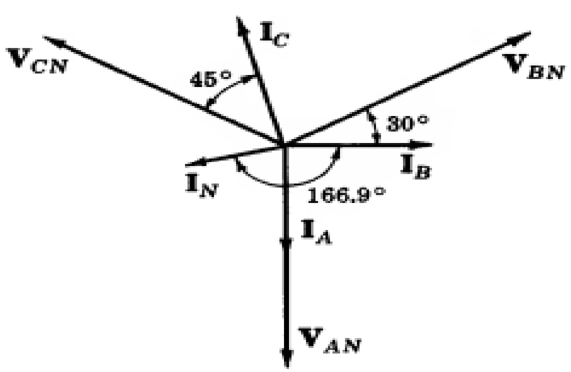
Example 5.

A three-phase, four-wire, 208 volt, *CBA* system has a wye-connected load with $\mathbf{Z}_A = 6\underline{/0^\circ}$, $\mathbf{Z}_B = 6\underline{/30^\circ}$ and $\mathbf{Z}_C = 5\underline{/45^\circ}$. Obtain the three line currents and the neutral current. Draw the phasor diagram.



$$\mathbf{I}_A = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A} = \frac{120\underline{/-90^\circ}}{6\underline{/0^\circ}} = 20\underline{/-90^\circ}$$

$$\mathbf{I}_B = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B} = 20\underline{/0^\circ} \qquad \mathbf{I}_C = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C} = 24\underline{/105^\circ}$$



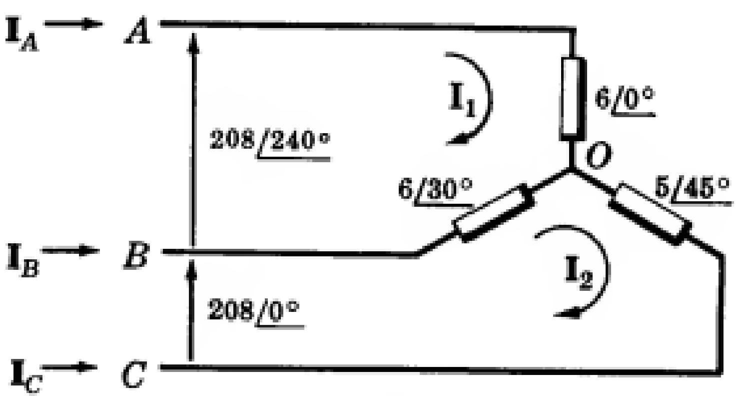
$$\mathbf{I}_N = -(\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C) = -(20\underline{/-90^\circ} + 20\underline{/0^\circ} + 24\underline{/105^\circ}) = 14.1\underline{/-166.9^\circ}$$

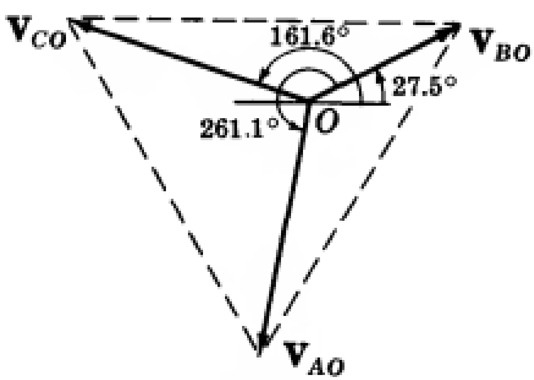
UNBALANCED THREE-WIRE, WYE-CONNECTED LOAD

- The common point of the three load impedances is not at the potential of the neutral and is marked "O" instead of N.
- The voltages across the three impedances can vary considerably from line to neutral magnitude, as shown by the voltage triangle which relates all of the voltages in the circuit.

Example 6.

A three-phase, three-wire, 208 volt, *CBA* system has a wye-connected load with $\mathbf{Z}_A = 6\angle 0°$, $\mathbf{Z}_B = 6\angle 30°$ and $\mathbf{Z}_C = 5\angle 45°$. Obtain the line currents and the phasor voltage across each impedance. Construct the voltage triangle and determine the displacement neutral voltage, $\mathbf{V}_{ON}$.

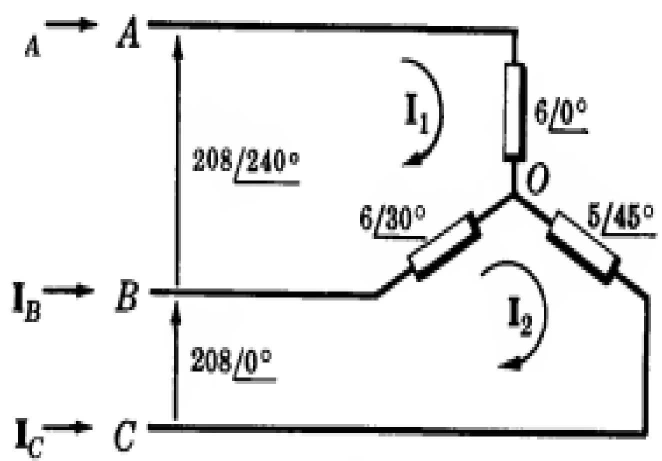




- Draw the circuit diagram and select mesh currents as shown in Fig.
- Write the corresponding matrix equations (Crammer Rule)

UNBALANCED THREE-WIRE, WYE-CONNECTED LOAD

$$\begin{bmatrix} 6\angle 0^\circ + 6\angle 30^\circ & -6\angle 30^\circ \\ -6\angle 30^\circ & 6\angle 30^\circ + 5\angle 45^\circ \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} 208\angle 240^\circ \\ 208\angle 0^\circ \end{bmatrix}$$



$$\mathbf{I}_1 = 23.3\angle 261.1^\circ \qquad \mathbf{I}_2 = 26.5\angle -63.4^\circ$$

$$\mathbf{I}_A = \mathbf{I}_1 = 23.3\angle 261.1^\circ$$

$$\mathbf{I}_B = \mathbf{I}_2 - \mathbf{I}_1 = 26.5\angle -63.4^\circ - 23.3\angle 261.1^\circ$$
$$= 15.45\angle -2.5^\circ$$

$$\mathbf{I}_C = -\mathbf{I}_2 = 26.5\angle 116.6^\circ$$

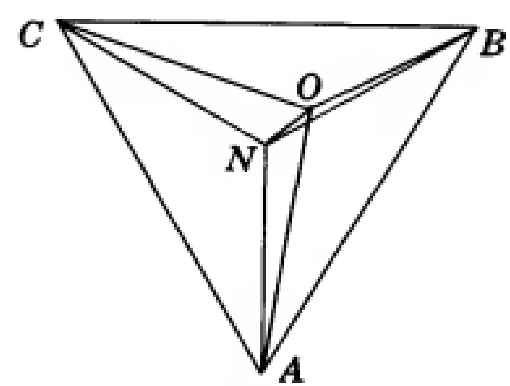
Now the voltages across the three impedances are given by the products of the line currents and the corresponding impedances.

$$\mathbf{V}_{AO} = \mathbf{I}_A\mathbf{Z}_A = 23.3\angle 261.1^\circ\,(6\angle 0^\circ) = 139.8\angle 261.1^\circ$$

$$\mathbf{V}_{BO} = \mathbf{I}_B\mathbf{Z}_B = 15.45\angle -2.5^\circ\,(6\angle 30^\circ) = 92.7\angle 27.5^\circ$$

$$\mathbf{V}_{CO} = \mathbf{I}_C\mathbf{Z}_C = 26.5\angle 116.6^\circ\,(5\angle 45^\circ) = 132.5\angle 161.6^\circ$$

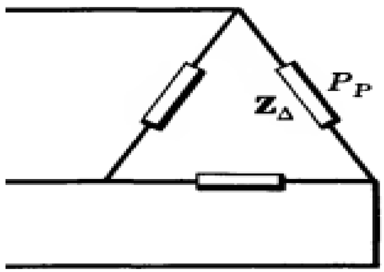
$$\mathbf{V}_{ON} = \mathbf{V}_{OA} + \mathbf{V}_{AN} = -139.8\angle 261.1^\circ + 120\angle -90^\circ$$
$$= 28.1\angle 39.8^\circ$$

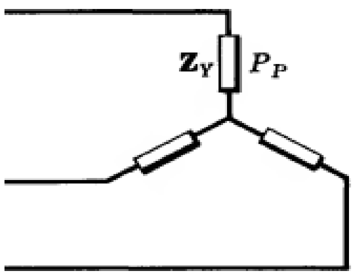


$$\mathbf{V}_{ON} = \frac{\mathbf{V}_{AN}\mathbf{Y}_A + \mathbf{V}_{BN}\mathbf{Y}_B + \mathbf{V}_{CN}\mathbf{Y}_C}{\mathbf{Y}_A + \mathbf{Y}_B + \mathbf{Y}_C}$$

POWER IN BALANCED THREE-PHASE LOADS

➢ Since the phase impedances of balanced wye or delta loads contain equal currents, the phase power is one-third of the total power.





- The voltage across is line voltage
- The current is phase current.
- The angle between V & I is the angle on the impedance.

Phase power $P_P = V_L I_P \cos\theta$

Total power $P_T = 3 V_L I_P \cos\theta$

For a balanced Δ-connected loads:

$$I_L = \sqrt{3}\, I_P$$

$$P_T = \sqrt{3}\, V_L I_L \cos\theta$$

- The voltage across is phase voltage
- The current is line current.
- The angle between V & I is the angle on the impedance.

Phase power $P_P = V_P I_L \cos\theta$

Total power $P_T = 3 V_P I_L \cos\theta$

For a balanced Y-connected loads:

$$V_L = \sqrt{3}\, V_P$$

$$P_T = \sqrt{3}\, V_L I_L \cos\theta$$

POWER IN BALANCED THREE-PHASE LOADS

$$P_T = \sqrt{3}\, V_L I_L \cos\theta$$

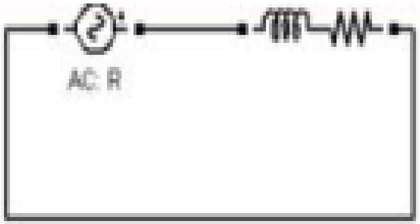
$$S_T = \sqrt{3}\, V_L I_L$$

$$Q_T = \sqrt{3}\, V_L I_L \sin\theta$$

INSTANTANEOUS THREE-PHASE POWER

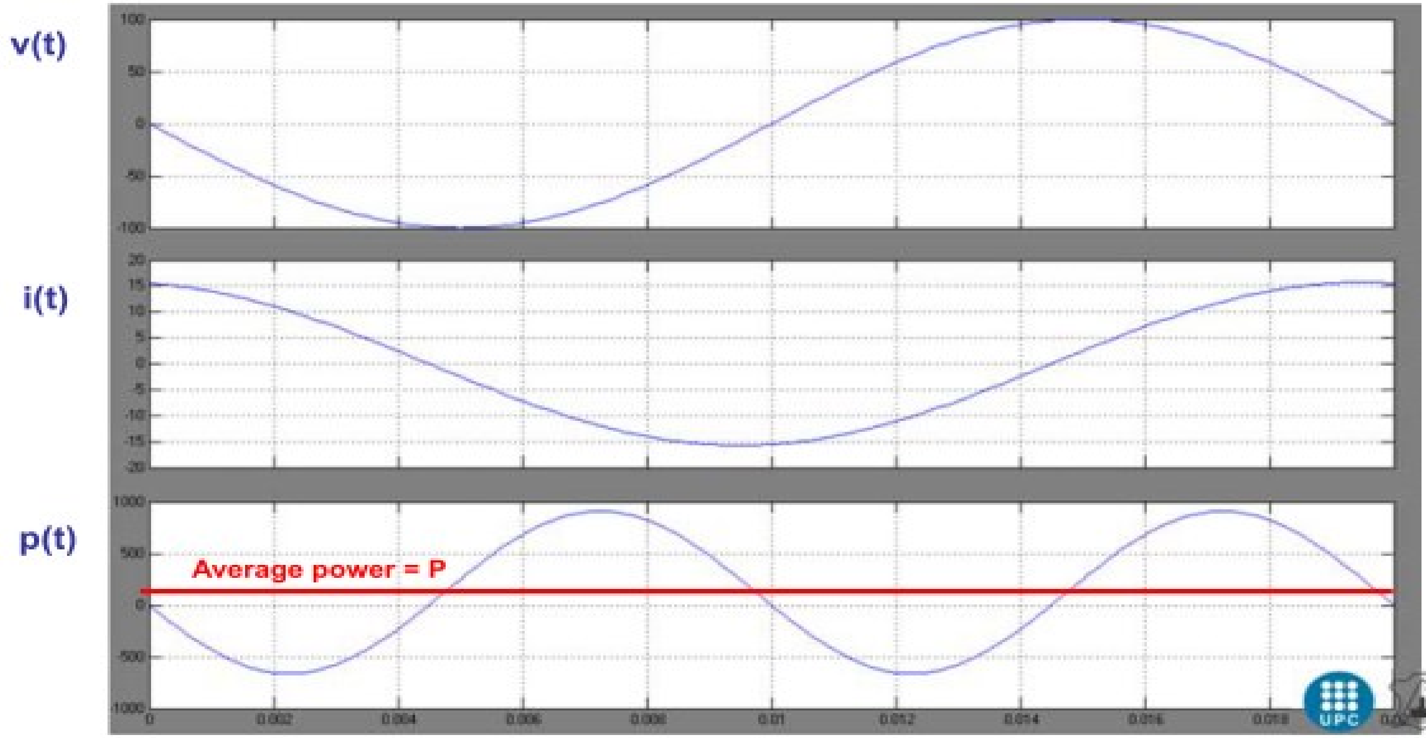
➢ Remember: The instantaneous Single-phase power

$$p(t) = v(t)\cdot i(t) = V_0\cdot\cos(w\cdot t + \varphi_V)\cdot I_0\cdot\cos(wt + \varphi_I)$$



$$\cos A\cdot\cos B = 0{\cdot}5\cdot[\cos(A+B) + \cos(A-B)]$$

$$p(t) = \underbrace{1/2\cdot V_0\cdot I_0\cdot\cos(\varphi_V - \varphi_I)}_{\text{Constant}} + \underbrace{1/2\cdot V_0\cdot I_0\cdot\cos(2wt + \varphi_V + \varphi_I)}_{\text{Oscillates at twice the mains frequency!}} \text{ watt}$$



INSTANTANEOUS THREE-PHASE POWER

The instantaneous 3-phase power

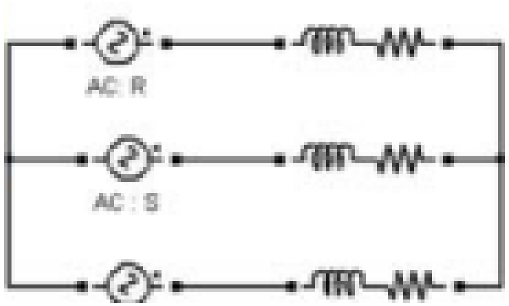
$$p(t) = V_{AN}\, Ia + V_{BN}\, Ib + V_{CN}\, IC$$

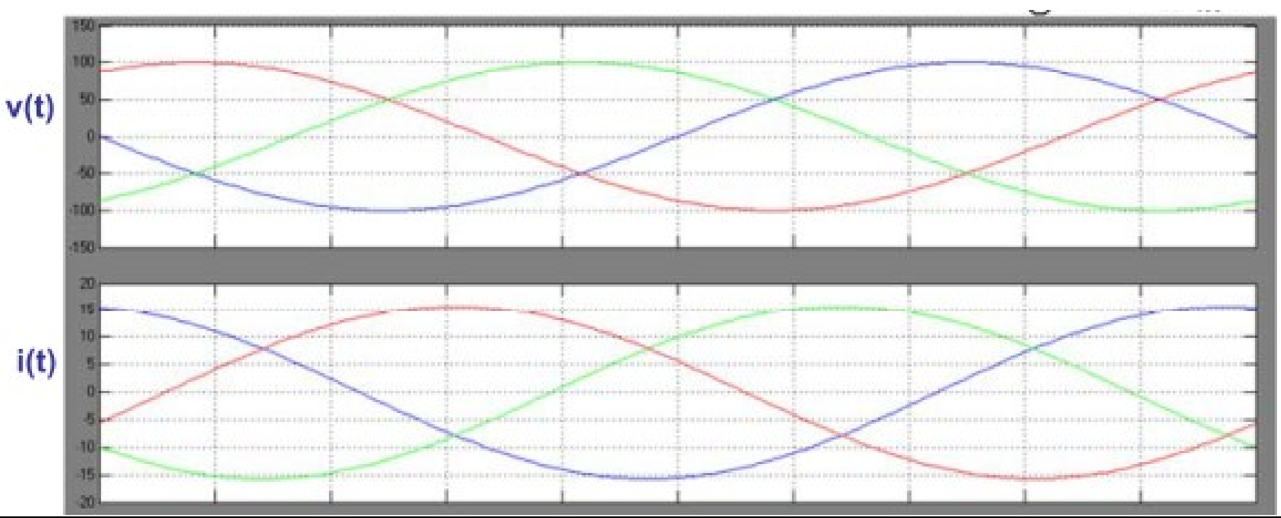
$$= \sqrt{2}V_p \cdot \cos(wt + \varphi_V) \cdot \sqrt{2}I_p \cdot \cos(wt + \varphi_I)$$

$$+ \sqrt{2}V_p \cdot \cos(wt - 120^\circ + \varphi_V) \cdot \sqrt{2}I_p \cdot \cos(wt - 120^\circ + \varphi_I)$$

$$+ \sqrt{2}V_p \cdot \cos(wt + 120^\circ + \varphi_V) \cdot \sqrt{2}I_p \cdot \cos(wt + 120^\circ + \varphi_I)$$

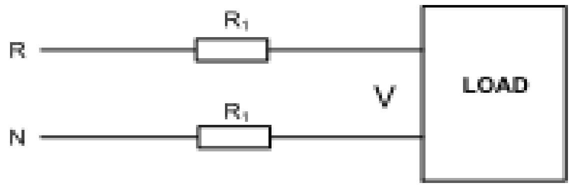
Assignment:
Complete the steps:
(mathematically/Using Multisim/using Matlab)
to find the instantaneous 3-phase power





POWER LOSSES: THREE-PHASE/SINGLE PHASE

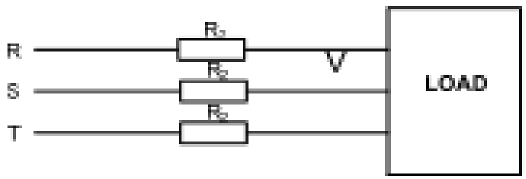
Single-phase line



$$I = \frac{P_{load}}{V \cdot \cos\varphi}$$

$$P_{losses} = 2 \cdot R_1 \cdot I^2 = 2 \cdot R_1 \cdot \frac{P_{load}^2}{V^2 \cdot \cos^2\varphi}$$

Three-phase line



$$I = \frac{P_{load}}{\sqrt{3} \cdot V \cdot \cos\varphi}$$

$$P_{losses} = 3 \cdot R_2 \cdot I^2 = 3 \cdot R_2 \cdot \frac{P_{load}^2}{(\sqrt{3})^2 . V^2 \cdot \cos^2\varphi} = R_2 \cdot \frac{P_{load}^2}{V^2 \cdot \cos^2\varphi}$$

Supposing same losses

$$2R_1 = R_2 \rightarrow 2\rho \frac{l}{S_{1p}} = \rho \frac{l}{S_{3p}} \rightarrow S_{3p} = \frac{1}{2} S_{1p}$$

Single-phase line: 2 conductors of length l and section S_{1p}

Three-phase line: 3 conductors of length l and section $S_{3p} = 1/2 S_{1p}$

As a result: $\text{weight}_{3p\text{-cables}} = 3/4\,\text{weight}_{1p\text{-cables}}$

$$\frac{V_{3p}}{V_{1p}} = \frac{3 * (l * S_{3p})}{2 * (l * S_{1p})} = \frac{3 * (S_{1p}/2)}{2 * (S_{1p})} = \frac{3}{4}$$