DESIGN SYNTHESIS FOR SINGLE- AND MULTI-BAY STEEL FRAMES ACCORDING TO ECP’ 01

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ABSTRACT

Minimum weight design of single-and multi-bay steel portal frames is presented. The design variables are the dimensions of prismatic built-up sections for beams and columns. Design constraints are considered as per ECP’ 01: shape, buckling, stresses, and deflection constraints. Both compact and non-compact sections are included in the formulation. Cases of loading comprise vertical and lateral loads. Analysis is done using Displacement Stiffness Method. An optimization technique based on the Method of Feasible Directions -through an implicit formulation– is adopted. Several examples are presented in order to assess the advantages of adopting optimization in structural steel design as compared to other classical design approaches.

Key words: Multi-bay; Steel; Frames; Optimization; Compact; Non-compact; Prismatic; Built-up Sections.

1. INTRODUCTION

Portal frames are one of the most important structures in the family of steel industrial buildings. Consequently, the minimization of the design weight had been a legitimate goal for the designers and researchers for the last few decades. Camp et al. [1] developed an Ant Colony Optimization (ACO) technique for discrete optimization of steel frames. The constraints considered were the serviceability and strength requirements as specified by AISC-LRFD. Optimization of steel frames under seismic loading was studied by Moharramy and Alavinasab [2]. The constraints included limits on stresses, deflections, side sways, inter-story drifts and upper and lower bounds on member sizes according to AISC-ASD. Sarma and Adeli [3] created discrete multi-criteria optimization model for design of large steel structures. The model was used to perform a comparative study of optimum design of steel high-rise building structures using AISC-ASD and AISC-LRFD. Schinler [4] developed an optimization algorithm to design fully restrained and partially restrained steel frames. He selected an...
evolutionary algorithm to stochastically guide the algorithm through the solution space of available designs and arrive at an evolved frame. A method of advanced analysis was used to assess the adequacy of the steel frames in lieu of design specification and code requirements. Pezeshk et al. [5] presented a genetic algorithm-based optimization procedure for design of nonlinear steel frames. They used the genetic algorithms as a tool to achieve discrete nonlinear optimal or near-optimal designs in accordance with the requirements of the AISC-LRFD specification. Saadoun and Arora [6] described a practical formulation for optimum design of framed structures under multiple loading and constraint conditions. An interactive software system for AISC code limits on element stresses, member maximum deflection, stability and slenderness ratios, width thickness ratio, and nodal displacements were imposed in the design process.

This research aims at developing an efficient optimization formulation for single-and multi-bay steel portal frames according to the latest version of the Egyptian Code of Practice for steel construction ECP’01 [7]. In order to optimize a wide spectrum of frames, the Displacement Stiffness Method for structural analysis [8] is used as an appropriate analysis tool. The argument remains valid for the Method of Feasible Directions (MFD) optimization technique [9]. The algorithm is based on an implicit optimization formulation. It is equally applicable for compact and non-compact built-up prismatic sections. The next sections outline the optimization formulation, case studies, discussion, and the conclusions.

2. OPTIMIZATION PROBLEM

In this work, a generalized Displacement Stiffness Method is applied for both compact and non-compact sections. The input data include geometrical dimensions, support conditions, different cases of loading, and load combinations. The design variables are the cross-sectional dimensions, which include the web height and thickness as well as the flange width and thickness for each cross-section (see Fig. 1). The results include the optimized dimensions for each case of loading, value and location of maximum displacements, and frame weight for all iterations. The straining actions are computed for each joint of the member that has the same cross-section and then the maximum straining actions are utilized in the formulation. The optimization formulations for compact and non-compact sections are summarized hereafter.

Compact Sections

The constraints of compact sections according to ECP’01 may be divided into four groups: shape constraints (Eqs. 2-3), buckling constraints (Eqs. 4-7), stress constraints (Eqs. 8-9), and deflection constraints (Eqs. 10-11). It should be noted
that some of these constraints are applied at the cross-section level such as Eqs. (2, 3, 4, 8, and 9) while other constraints are applied at the member level such as Eqs. (5, 6, and 7). The constraints given by Equations 10 and 11 are applied at the overall frame structure level. The code equations can be re-written in the following optimization formulation:

Minimize:

\[ W_i = \sum_i A_i \times L_i \times \gamma_i \]  

(1)

Subject to:

\[ \frac{d_w}{t_w} - \frac{699 / \sqrt{F_y}}{13\alpha - 1} \leq 0 \quad \text{for } \alpha > 0.5 \]

or

\[ \frac{d_w}{t_w} - \frac{63.6 / \alpha}{\sqrt{F_y}} \leq 0 \quad \text{for } \alpha \leq 0.5 \]  

(2)

\[ C - \frac{15.3}{\sqrt{F_y}} \leq 0 \]  

(3)

\[ t_f - \frac{105}{\sqrt{F_y}} \leq 0 \]  

(4)

\[ L_a - \frac{20b_f}{\sqrt{F_y}} \leq 0 \]  

(5)

\[ L_a - \frac{1380A_f}{hF_y} C_b \leq 0 \]  

(6)

\[ \lambda_{\max} - 180 \leq 0 \]  

(7)
It should be mentioned here that all steel sections are designed as column-beam element and the webs of the sections are considered as unstiffened webs. Figure (2) shows the flowchart for the generation of constraints for the case of compact sections as per ECP’01.

It is worthwhile noting that the buckling length factor ‘\( K \)’ for unbraced frames, which is given by the Eq. (13) is automatically generating in the program, rather using the alignment chart given in the code[10].

\[
\frac{G_A G_b (\pi/K)^2 - 36}{6(G_A + G_B)} - \frac{\pi(K)}{\tan(\pi/K)} = 0
\]  

**Lateral Torsional buckling**

When the compression flange is braced laterally at intervals exceeding \( L_u \) that defined by Eqs. (5 and/or 6), the allowable bending stresses in compression, \( F_{bc} \), will be taken as the larger value from the following equations:

For \( \frac{t_f L_u}{b_f \, d} > 10 \), then

\[
F_{bb1} = \frac{800}{L_u \, d} \frac{C_b}{A_f} \leq 0.58 F_y
\]  

For \( \frac{t_f L_u}{b_f \, d} < 0.4 \) there are three cases as follows:

(i) For \( \frac{L_u}{r_f} \leq 84 \sqrt{\frac{C_b}{F_y}} \), then

\[
F_{bb2} = 0.58 F_y
\]  

\[
\frac{f_{cu}}{F_c} + \frac{f_{bcx}}{F_{bcx}} A_f - 1.0 \leq 0 \tag{8}
\]

\[
\frac{Q_{\text{max}}}{d t_w} - q_b \leq 0 \tag{9}
\]

\[
\delta_y - \frac{L}{300} \leq 0 \tag{10}
\]

\[
\delta_y - \frac{H}{150} \leq 0 \tag{11}
\]

\[
0 \leq h_w, t_w, b_f, t_f \leq 100000 \tag{12}
\]
(ii) For \( \frac{84 \sqrt{C_b}}{F_y} \leq L_u \leq 188 \sqrt{C_b} \), then
\[
F_{lbb} = (0.64 - \frac{L_u}{1.176 \times 10^5 \times C_b})F_y \leq 0.58F_y
\]  
\[
(16)
\]

(iii) For \( \frac{L_u}{r_f} \geq 188 \sqrt{\frac{C_b}{F_y}} \), then
\[
F_{lbb} = \frac{12000}{L_u}C_b \leq 0.58F_y
\]  
\[
(17)
\]

Alternatively, the more accurate value of lateral torsional buckling stress may be computed as follows:
\[
F_{bb} = \sqrt{F_{lbb1}^2 + F_{lbb2}^2} \leq 0.58F_y
\]  
\[
(18)
\]

**Non-compact sections**

In the case of non-compact section, according to ECP’01, Equations 2 and 3 will be:
\[
d_w \leq \frac{190}{\sqrt{F_y}} \frac{1}{2 + \psi}
\]  
\[
(19)
\]
\[
\frac{C}{t_f} \leq \frac{21}{\sqrt{F_y}}
\]  
\[
(20)
\]

where \( \psi = (\frac{-N}{A} + \frac{M}{I_y})/F_y > -1 \)

The previous equations can be re-written in the following optimization formulation:

**Minimize:**

\[
W_l = \sum_{i}^{n} A_i \times L_i \times y_i
\]  
\[
(21)
\]
Fig. 2. Flowchart for Generation of Constraints for Compact Sections
Fig. 2. Flowchart for Generation of Constraints for Compact Sections (Contd.)
Fig. 2. Flowchart for Generation of Constraints for Compact Sections (Contd.)
Fig. 2. Flowchart for Generation of Constraints for Compact Sections (Concluded)
3. APPLICATIONS AND CASE STUDIES

Four examples of steel frame structures are presented. The examined frames have spans of 2200, 2400 and 2500 cms. Different support conditions are included. Single and multi-bay frames with normal mild steel and high tensile steel are investigated.

Example 1

This frame is a two-hinged frame given in reference [11]. The span of the frame \( L \) is 2200 cm, the height \( H \) is 600 cm, and the angle of inclination \( \phi \) of the rafter is 5.7° (refer to Fig. 3). Steel grade is normal mild steel (24/37) and the live and wind loads are considered according to ECP’93 [12]. The design of the frame is done using a classical approach, which gives prismatic hot-rolled cross-sections (B.F.I.B. No. 28) and frame weight of 3.84 tons.
Fig. 3. Layout of Frame for Example 1

To describe the advantages of optimum design as compared to above-mentioned classical approach, the frame is investigated using the algorithm of compact sections. An optimum weight of 2 tons is obtained after 595 iterations and five seconds, with an improvement of 48%. In order to demonstrate the efficiency of the implicit formulation, three other different starting points with weights 321.3, 80.3 and 26.77 tons are studied. A minimum weight of 2 tons is reached after 595, 405, and 479 iterations, respectively. Table (1) gives the starting and final optimal cross-sectional dimensions for these starting points. The iteration histories for the objective functions are shown in Fig. (4). The shape constraints, combined stresses for both column and rafter (Eqs. 2, 3, 4 and 8) are the active constraints. Whereas, the lateral buckling constraint of Eq. (6) is deactivated due to the oscillation that occurs when it is activated. In any case, this constraint is not active at optimally.

Table 1. Design History for Example 1

<table>
<thead>
<tr>
<th>Section Type</th>
<th>Starting Point No.</th>
<th>Cross-Sectional Dimensions (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Starting</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( t_w ) ( h_w ) ( t_f ) ( b_f )</td>
</tr>
<tr>
<td>Column</td>
<td>(1)</td>
<td>20 200 20 200 0.7 48.7 1.0 21.2</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>10 100 10 100 0.7 48.6 1.0 21.3</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>5 100 5 50 0.7 46.6 1.0 22.0</td>
</tr>
<tr>
<td>Rafter</td>
<td>(1)</td>
<td>20 200 20 200 0.8 53.8 0.8 19.1</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>10 100 10 100 0.8 52.3 0.9 19.7</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>5 100 5 50 0.8 52.4 0.9 19.5</td>
</tr>
</tbody>
</table>
Example 2

The 2200 cm span frame analyzed previously in Example (1) is optimized again using the algorithm for non-compact sections. Starting with an overdesign of 8 tons as a starting point, an optimum weight of 1.57 tons is obtained after 144 iterations and three seconds. Shape, combined stresses, and vertical deflection given by Eqs. (22, 23, 24, 26, and 28) are still the active constraints. The starting and final optimal sectional dimensions are given in Table (2). The iteration history is shown in Fig. (5).

Table 2. Design History for Example 2

<table>
<thead>
<tr>
<th>Section Type</th>
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<td>4</td>
</tr>
<tr>
<td>Rafter</td>
<td>4</td>
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</tbody>
</table>
Example 3

To show the robustness of the present formulation, this example is chosen from previous work conducted by other researchers. The pitched roof frame given in Ref. [13] is selected for this demonstration. The fixed frame span $L$ is 2400 cms, the height $H$ is 600 cms, and angle of rafter slope is $11.3^\circ$. It consists of tapered I-section members with constant flange width, flange thickness and web thickness. The frame is subjected to a vertical live load 2 t/m and steel grade 44 is used (see Fig. 6).

$$L.L. = 2 \, t/m$$

Fig. 5. Iteration History for Example 2

Fig. 6. Layout of Frame for Example 3
In the cited reference, an algorithm based on an optimality criteria technique is used. The web height is considered as a unique design variable to avoid the calculation of large sets of Lagrange Multipliers. The tapered member is designed according to LRFD [14], and then the method is modified and developed according to the ECP’89 [15].

Fig. 7. Section Results for Frame of Example 3

The displacements at joints and combined axial and flexural strength are taken as constraints. Moreover, the deflection constraint is modified – in the cited - to L/160 (Eq. 10 or 28 in this work). A total weight of 4.25 tons is obtained. Under the same conditions, loads and ECP’89 code, and using an unrealistic starting point with total weight of 42.49 tons, but utilizing prismatic members, a minimum weight of 4.33 tons is obtained after 170 iterations, which is 1.8 % greater than that given in the cited reference using tapered members. According to the optimization results, the combined stresses constraint is dominant in the design (Eq. 26 in this work). This is due to the excessive live load and the rafter steep inclination of the investigated frame. Another starting point is used and the results are presented in Table (3) and Fig. (8). The frame is re-optimized according to ECP’01 and the same results are obtained.

It should be mentioned here that ECP’89 – which is used to solve this example in the stated reference - and the latest code version ECP’01 are different in the shape and buckling constraints. However, the combined stresses constraint is the
Table 3. Design History for Example 3

<table>
<thead>
<tr>
<th>Section Type</th>
<th>Starting Point No.</th>
<th>Cross-Sectional Dimensions (cm)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Starting</td>
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<td>$h_w$</td>
</tr>
<tr>
<td>Column</td>
<td>(1)</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>1.5</td>
<td>80</td>
</tr>
<tr>
<td>Rafter</td>
<td>(1)</td>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>1.5</td>
<td>80</td>
</tr>
</tbody>
</table>

Fig. 8. Iteration History for Example 3

same in both versions of the code. Shape constraints are not taken into consideration in this example and buckling constraints do not affect the optimal design. The same final weight is therefore, obtained when the frame is redesigned according to ECP’01.

Example 4

Figure 9 shows a two gable-pitched frame with fixed bases. This frame is constructed in Dahran Airport, Kingdom of Saudi Arabia. The span $L$ of each bay is 2500 cms, the height $H$ is 800 cms, and the inclination of the rafter was 1:10. The frame is designed with prismatic columns and tapered rafters loads are considered according to ECP’01. High strength steel (36/52) is used. The
dimensions of the built-up sections are shown in the figure. Both outer and intermediate columns have the same cross-sections. The total weight of the constructed frame is 3.65 tons.

Using three starting points of weights 46.63, 24.8, and 6.41 tons, and considering the case of loading in which one bay is loaded by total loads, and the second bay is loaded by dead load only, an optimal solution of 3.78 tons is obtained. In this solution, different cross-sections are used for outer columns, intermediate column and rafters. The dimensions of these sections are shown in Fig. (9). The active constraints are shape constraints and combined stresses for outer columns and rafters (Eqs. 22, 23, 24, and 26), lateral buckling constraint for intermediate column (Eq. 25), and vertical deflection (Eq. 28). The calculated results are presented in Table (4) and Fig. (10). Iteration histories are shown in Fig. (11), the case where the two bays are subject to dead and live loads is also considered, a minimum weight of 3.52 tons is obtained. Fig. (11) represents the final sections dimensions for this case.
Table 4. Design History for Example 4

<table>
<thead>
<tr>
<th>Section Type</th>
<th>Starting Point No.</th>
<th>Cross-Sectional Dimensions (cm)</th>
<th></th>
<th></th>
<th></th>
<th>Starting</th>
<th>Final</th>
<th>Constructed frame</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>t_w</td>
<td>h_w</td>
<td>t_f</td>
<td>b_f</td>
<td>t_w</td>
<td>h_w</td>
<td>t_f</td>
</tr>
<tr>
<td>Edge Columns</td>
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<td>100</td>
<td>10</td>
<td>100</td>
<td>0.63</td>
<td>63.07</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>2</td>
<td>90</td>
<td>3</td>
<td>40</td>
<td>0.63</td>
<td>63.07</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>1</td>
<td>50</td>
<td>2</td>
<td>15</td>
<td>0.63</td>
<td>62.80</td>
<td>0.61</td>
</tr>
<tr>
<td>Intermediate Column</td>
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<td>100</td>
<td>10</td>
<td>100</td>
<td>0.66</td>
<td>66.18</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>2</td>
<td>90</td>
<td>3</td>
<td>40</td>
<td>0.66</td>
<td>66.18</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
<td>1</td>
<td>50</td>
<td>2</td>
<td>15</td>
<td>0.68</td>
<td>67.79</td>
<td>0.84</td>
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<tr>
<td>Rafters</td>
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<td>100</td>
<td>10</td>
<td>100</td>
<td>0.67</td>
<td>67.25</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>(2)</td>
<td>2</td>
<td>90</td>
<td>3</td>
<td>40</td>
<td>0.67</td>
<td>67.25</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>(3)</td>
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<td>50</td>
<td>2</td>
<td>15</td>
<td>0.66</td>
<td>65.77</td>
<td>1.02</td>
</tr>
</tbody>
</table>

Fig. 11. Iteration History for Example 4
4. CONCLUSIONS

An efficient algorithm is developed for design synthesis of single- and multi-bay steel frames subject to vertical and lateral loads. In this work, the frames are analyzed using the Displacement Stiffness Method. The optimization technique based on the Method of Feasible Directions – through an implicit formulation - is adopted. The design variables are the dimensions of prismatic built-up sections for beams and columns. All ECP’01 constraints for shape, buckling, stresses, and deformations are incorporated. Both compact and non-compact sections are included in the formulation. The objective function is represented by the total weight of the frame. Four examples are presented to demonstrate the robustness and validity of the formulation. The obtained results show the efficiency, practicality, and versatility of the adopted optimization over other classical design approaches.

NOTATIONS

The following symbols are used in this work:

- $A_f$: Cross-sectional area of flange.
- $b_f$: Total flange width of the section.
- $C$: Outstanding flange width.
- $C_b$: Code coefficient, ECP’01, Eq. (2.28) & Table (2.2).
- $d_w$: Total height of the web.
- $f$: Difference of frame height at column and at mid-span (ridge).
- $f_{bcx}$: Actual compressive bending stress based on moments about x-axis.
- $f_{cax}$: Actual compressive stress due to axial compression.
- $F_c$: Allowable stress in axial compression.

Fig. 12. Section Results for Example 4
$F_{cb}$ Allowable stress in bending.
$F_E$ Euler stress.
$FM,FM\text{MAX}$ Bending moment and maximum bending moment.
$FN,FN\text{MAX}$ Normal force and maximum normal force.
$FQ,FQ\text{MAX}$ Shear force and maximum shear force.
$F_{ltb1},F_{ltb2}$ Lateral torsional buckling stress.
$F_Y$ Yield stress of steel.
$h$ Total height of section.
$H$ Column height.
$ISEC$ Number of a section.
$K_b$ Buckling length factor.
$K_q$ Buckling factor for shear.
$L$ Frame span.
$L_u$ Effective laterally unsupported length of the compression flange.
$MEMN$ Number of a member.
$NM$ Number of members of frame.
$NS$ Number of sections of frame.
$Q_{\text{max}}$ Maximum shear force.
$q_b$ Buckling shear stresses.
$r_T$ Radius of gyration about minor axis of section comprising flange plus sixth of the web area.
$S$ Rafter length.
$S_w$ Size of weld.
$S_p$ Spacing between purlins.
$tw$ Thickness of web.
$W_t$ Total weight of frame.
$\alpha_1, \alpha_2$ Code factor, ECP’01, Table 2.1a.
$\gamma$ Specific weight of steel.
$\lambda$ Web slenderness parameter.
$\lambda_{\text{max}}$ Maximum slenderness ratio.
$\delta, \delta_V$ Maximum vertical deflection due to live load.
$\delta_H$ Maximum horizontal deflection due to live load.
$\phi$ Slope angle of the rafter.

REFERENCES