(1) The operating and the maintenance expense of a machine are expected to increase 0.5% for month. This month's expenses are $200. Find the equal monthly series that is equivalent to the monthly expense over 5-years for an interest rate of 21% compounded quarterly.

\[ i/q = i_{ncq} / m \]
\[ i/q = 21/4 = 0.525 / q \]
\[ i_{eff} = (1 + i / q)^m - 1 \]
\[ i_{eff} = (1 + 0.0525)^4 - 1 = 0.227123 \text{ year} \]
\[ i/\text{month} = (1 + 0.227123)^{1/12} - 1 = 0.017209 / \text{month} \]
\[ p = A \left[ \frac{1 - (1 + g)^n}{i - g} \right] = 200 \left[ \frac{1 - (1 + .005)^{60}}{.0172 - .005} \right] \]
\[ p = 200(42.2277) = 8445.53 \]

\[ i/\text{month} = 0.0172 / \text{month} \]
\[ n = 60 \text{ month} \]
\[ A = P(A/P, i, n) \]
\[ A = 8445.53 \left[ \frac{i(1+i)^n}{(1+i)^n-1} \right] = 8445.53 \left[ \frac{.0172(1+.0172)^{60}}{(1+.0172)^{60} - 1} \right] = 226.773 \]
(2) A land surveyor just starting in private practice needs a van to carry crew and equipment. He can lease a used van for $3000 per year, paid at the beginning of each year in which case maintenance is provided. Alternatively, he can buy a used van for $7000, and pay for maintenance himself. He expects to keep the van three years at which time he could sell it for $1500. **What is the most he should pay for uniform annual maintenance to make it worthwhile buying the van instead of leasing it.** If the MARR is 20%.

\[
EUAC(\text{Lease}) = [3000(P/A, 20\%, 2) + 3000](A/P, 20\%, 3)
\]
\[
EUAC(\text{Lease}) = [3000(1.528) + 3000](0.4747)
\]
\[
EUAC(\text{Lease}) = 3600.125
\]
\[
EUAC(\text{Buy}) = [7000 + M(P/A, 20\%, 3) - 1500(P/F, 20\%, 3)](A/P, 20\%, 3)
\]
\[
EUAC(\text{Buy}) = [7000 + M(2.106) - 1500(0.5787)](0.4747)
\]
\[
= 3322.9 + .9997M - 412.06 = .9997M + 2910.84
\]
\[
EUAC(\text{Buy}) \leq EUAC(\text{Lease})
\]
\[
\therefore M = 689.49
\]
(3) Materials testing machine was purchased for $20,000 and was to be used for 5 years with an expected residual salvage value of $5000. Graph the annual depreciation charges and year-end book values obtained by using:

a) sum–of–digits  
b) Double declining–balance depreciation.

**SOLUTION**

**a) Sum–of–digits**

\[ soy \quad for \quad any \quad J \quad year = \frac{(p-s)}{N} \left[ N - J + 1 \right] \]

\[ N/2 \times (N+1) = 15 \]

\[ P - S = 20,000 - 5,000 = 15,000 \]

\[ soy \quad for \quad any \quad J \quad year = \frac{15,000}{15} \left[ N - J + 1 \right] \]

\[ BV = P - \sum \frac{P-S}{N/2(N+1)} \left[ N - J + 1 \right] \]

<table>
<thead>
<tr>
<th>End of Year</th>
<th>SOY Dep.</th>
<th>B.V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5000</td>
<td>20000-5000=15000</td>
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<tr>
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<td>1100</td>
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<td>2000</td>
<td>6000</td>
</tr>
<tr>
<td>5</td>
<td>1000</td>
<td>5000</td>
</tr>
</tbody>
</table>
b) Double declining – balance depreciation

\[ DDBD \text{ for any } J \text{ year } = \frac{2P}{N}(1-2/N)^{J-1} \]

\[
\frac{2 \times 20000}{5} = 8000 \\
2/5 = 0.4 \\
B.V = P(1-2/N)^J
\]

<table>
<thead>
<tr>
<th>End of Year</th>
<th>DDB Dep.</th>
<th>B.V</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>20000-8000=12000</td>
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<td>2</td>
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<td>4</td>
<td>1728</td>
<td>2592</td>
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<tr>
<td>5</td>
<td>1036.8</td>
<td>1555.2</td>
</tr>
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</table>

(4) Woman deposited $10000 into an account at her credit union. The money was left on deposit for 10 years. During the first five years the woman earned 15% interest (nominal), compounded monthly. The credit union then changed it’s interest policy so that the second five years the woman earned 18% interest (nominal) compounded quarterly.

- How much money was in the account at the end of the 10 years?
- Calculate the nominal interest rate compounded annually that the woman received.

**Solution:**

**Part a:**
At the end of 5 years:

\[ i\% = \frac{15}{12} = 1\frac{1}{4} \% \text{ per month} \]

\[ n = 5 \times 12 = 60 \text{ month} \]
At the end of 10 years:

\[ i\% = \frac{18}{4} = \frac{1}{2} \% \text{ per quarter} \]

\[ n = 5 \times 4 = 20 \text{ quarter} \]

\[ Ft = F1(F / P, i, n) = 21070(F / P, 1.5\%, 20) = 21070(2.412) = 50820.84 \]

Part b:

\[ Ft = P(F / P, i, n) = 50820.84 \]

\[ 10000(F / P, 1.10) = 50820.84 \]

\[ (F / P, 1.10) = 5.082084 \]

\[ \text{assume } i = 15\% \quad (F / P, 15\%, 10) = 4.046 \]

\[ \text{assume } i = 18\% \quad (F / P, 18\%, 10) = 5.234 \]

\[ \text{interpolate: } i = 15\% + (18\% - 15\%)
\frac{4.046 - 5.082}{4.046 - 5.234} = 17.616\% / \text{years} \]