Answer all of the following questions

Question 1 (13)

A. Answers:
   i. Area III is the feasible region
   ii. Point D is optimal
   iii. Constraints 2 and 3 are binding
   iv. $S_2$ and $S_3$ are equal to 0

B. The graph is shown,
   $X = 2$, $Y = 1$, and $Z = 4$
   Yes, there is a redundant constraint; $Y \leq 1$

Question 2 (10)

a) $x_1 = 13.33$, $x_2 = 10$, $x_3 = 0$, $s_1 = 0$, $s_2 = 0$, $s_3 = 23.33$, $z = 763.33$ [2 marks]
b) Constraints 1 and 2 are binding? [2 marks]
c) Dual prices: -0.778, 5.556, and 0. They are the amount of improvement in Z if the RHS increases by one unit [2 marks]
d) The value of the objective function would increase by 40 [2 marks]
e) The value of the objective function would decrease by 7.78. [2 marks]

Question 3 (10)

X1: number of Speedhawk boats to purchase
X2: number of Silverbird boats to purchase
X3: number of Catman boats to purchase
X4: number of classy boats to purchase

Objective function: maximize the total daily profits:
Max $Z = 70X_1 + 80X_2 + 50X_3 + 110X_4$

Budget limit constraint: $6,000X_1 + 7,000X_2 + 5,000X_3 + 9,000X_4 \leq 400,000$
At least 50 boats: $X_1 + X_2 + X_3 + X_4 \geq 50$
Same number from each supplier: $X_1 + X_2 = X_3 + X_4$
At least 200 capacity: $3X_1 + 5X_2 + 2X_3 + 6X_4 \geq 200$
Nonnegativity $X_i \geq 0$ $i = 1,2,3,4$

Complete model
Max $Z = 70X_1 + 80X_2 + 50X_3 + 110X_4$
S.T.
$6,000X_1 + 7,000X_2 + 5,000X_3 + 9,000X_4 \leq 400,000$
$X_1 + X_2 + X_3 + X_4 \geq 50$
$X_1 + X_2 = X_3 + X_4$
$3X_1 + 5X_2 + 2X_3 + 6X_4 \geq 200$
$X_i \geq 0$ $i = 1,2,3,4$

Question 4 (10)

A: # of model A copiers
B: # of model B copiers

Objective function: minimize total cost: Min $Z = 6,000A + 4,000B$
At least 6 machines: \[ A + B \geq 6 \]
At least one high-speed machine: \[ A \geq 1 \]
Copying capacity: \[ 20,000A + 10,000B \geq 75,000 \]
Integer and nonnegative: \[ A \text{ and } B \text{ are nonnegative integers} \]

Complete model:
Min \[ Z = 6,000A + 4,000B \]
S.T.
\[ A + B \geq 6 \]
\[ A \geq 1 \]
\[ 20,000A + 10,000B \geq 75,000 \]
\[ A \text{ and } B \text{ are nonnegative integers} \]

**Question 5 (10)**

\[ x_1 = \text{construction of a swimming pool} \rightarrow \text{binary 1 if constructed, 0 otherwise} \]
\[ x_2 = \text{construction of a tennis center} \rightarrow \text{binary 1 if constructed, 0 otherwise} \]
\[ x_3 = \text{construction of an athletic field} \rightarrow \text{binary 1 if constructed, 0 otherwise} \]
\[ x_4 = \text{construction of a gymnasium} \rightarrow \text{binary 1 if constructed, 0 otherwise} \]

Objective function: maximize total usage: Max \[ Z = 300x_1 + 90x_2 + 400x_3 + 150x_4 \]
Budget constraint: \[ 35,000x_1 + 10,000x_2 + 25,000x_3 + 90,000x_4 \leq 120,000 \]
Land constraint: \[ 4x_1 + 2x_2 + 7x_3 + 3x_4 \leq 12 \]
Swimming or tennis: \[ x_1 + x_2 \leq 1 \]
All variables are non negative binary

Complete model:
Max \[ Z = 300x_1 + 90x_2 + 400x_3 + 150x_4 \]
S.T.
\[ 35,000x_1 + 10,000x_2 + 25,000x_3 + 90,000x_4 \leq 120,000 \]
\[ 4x_1 + 2x_2 + 7x_3 + 3x_4 \leq 12 \]
\[ x_1 + x_2 \leq 1 \]
All variables are non negative binary

**Question 6 (7)**

Optimal path is SE – C – E – LN for a time of 11.3 hours