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1. Green valley mills produces carpet at plants in St. Louis and Richmond. The carpet is then ship to two outlets located in Chicago and Atlanta. The cost per ton of shipping carpet from each of two plants to the two warehouses is as follows:

From	To	
	Chicago	Atlanta
St. Louis	$٤٠	$٦٥
Richmond	٧٠	٣٠

The plant at St. Louis can supply 250 tons of carpet per week; the plant at Richmond can supply 400 tons per week. The Chicago outlet has a demand of 300 tons per week, and the outlet at Atlanta demands 350 tons per week.

- Formulate the problem as a linear programming model.
- The company wants to know the number of tons of carpet to ship from each plant to each outlet in order to minimize the total shipping cost. Solve this transportation problem using least cost method.

Solution (1)

- **Formulate the problem as a LP model.**

$$Min \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} C_{ij} = 40X_{11} + 65X_{12} + 70X_{21} + 30X_{22}$$

Subjected to constrains

$$X_{11} + X_{12} = 250$$
$$X_{21} + X_{22} = 400$$
$$X_{11} + X_{21} = 300$$
$$X_{12} + X_{22} = 350$$
$$All \quad X_{ij} \geq 0$$

From	To		SUPPLY
	Chicago	Atlanta	
St. Louis	$٤٠	$٦٥	٢٥٠
Richmond	٧٠	٣٠	٤٠٠
DEMAND	٣٠٠	٣٥٠	٦٥٠

II. Using the Least- Cost method, find the basic feasible solution that would minimize the transportation cost.

Testing the problem is standard.

$$\sum_{i=1}^{2} a_i = 650$$

$$\sum_{j=1}^{2} b_j = 650$$

$$\therefore \sum_{i=1}^{2} a_i = \sum_{j=1}^{2} b_j$$

The Least- Cost method

	M1	M2	supplies
A	250 [40]	[65]	~~250~~ 0
B	50 [70]	350 [30]	~~400~~ ~~50~~ 0
Demands	~~300~~ ~~50~~ 0	~~350~~ 0	

The total cost of the transportation is given =250*40+50*70+350*30=24000

Number of basic feasible solution= m+n-1=2+2-1=3

III. Check the optimality for this solution.

	v1=40	v 2=0	supplies
u1=0	250 [40]	[65]	250
u2=30	50 [70]	350 [30]	400
Demands	300	350	

- **Used cell**

Cell (1,١) :u1+ v١=40
Cell (2,1) :u2+ v1=70
Cell (2,2) :u2+ v2=30

- **The optimality if** $\overline{C}_{ij} = C_{ij} - (U_i + V_j) \geq 0$ **for Unused cell**

$$\overline{C}_{13} = 65 - (0+0) = 65$$

The solution optimum

2.The Primo Insurance Co. is introducing two new product lines: special risk insurance and mortgages. The expected profit is $55 per unit of special risk insurance and $2 per unit on mortgages. Management wishes to establish sales quotas for the new product lines to maximize total expected profit. The work requirements are as follows:

Department	Work-hours per unit		Work-hours Available
	Special risk	Mortgage	
Underwriting	3	2	2400
Administration	0	1	800
Claims	2	0	1200

a. **Formulate a LP model for this problem.**

Solution (2)

Let Special risk = x

Mortgage =y

Objective function:

Maximize Z =55 x + 2y

Subject to the constraints:

$$3x+2y \leq 2400$$

$$Y \leq 800$$

$$2x \quad \leq 1200$$

$$x \text{ and } y \geq 0$$

1. Consider the following linear programming problem :

a) Max (5x1 + 6x_2)

Subject to the constraints:

$$4x_1 + 2x_2 \leq 420$$
$$2x_1 + 3x_2 \leq 360$$
$$X_1 \ , \ x_2 \geq 0$$

3. For the Hawkins Company, the monthly percentages of all that were received on time over the past 12months are as shown in table.

month	1	2	3	4	5	6	7	8	9	10	11	12
shipments	80	82	84,	83	83	84	85	84	82	83	84	83

2. Use a weight of .5 for the most recent observation, 1/3 for the second most recent, and 1/6 for the third most recent to compute a three –month weighted moving average for the time series.
3. What is the forecast for month 15?

Solution (3)

I -Three –month weighted moving average for the time series.

month	actual	weight	forecast
1	80	0.167	
2	82	0.333	
3	84	0.500	
4	83		82.67
5	83		83.17
6	84		83.17
7	85		83.5
8	84		
9	82		
10	83		
11	84		
12	83		

F4=80x.167+82x.333+84x.50=82.67

F5=82x.167+84x.333+83x.5=83.167

F6=84x.167+83x.333+83x.5=83.17

F7=83x.167+83x.333+84x.5=83.5

II - The forecast for month 15 using time trend series

Xi	Yi	Xi Yi	Xi^2
1	80	80	1
2	82	164	4
3	84	252	9
4	83	332	16
5	83	415	25
6	84	504	36
7	85	595	49
8	84	672	64
9	82	738	81
10	83	830	100
11	84	924	121
12	83	996	144
78	997	6502	650

sum Xi Yi=6502	sum Xi =78
sum Xi^2=650	sum Yi=997

$$y = mx + b$$

$$b = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} \quad = \frac{650*997 - 78*6502}{12*650 - 78*78} = 82.106$$

$$m = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} \quad = \frac{12*6502 - 78*997}{12*650 - 78*78} = 0.1503$$

$$y = 0.1503 * x + 82.106$$

$$y_{15} = 45.32 * 15 + 82.106 = 84.36$$

Solution (4)

a) Max ($5x1 + 6x_2$)

Subject to the constraints:

$$4x_1 + 2x_2 \leq 420$$
$$2x_1 + 3x_2 \leq 360$$

X_1 , $x_2 \geq 0$$4x_1 + 2x_2 \leq 420$

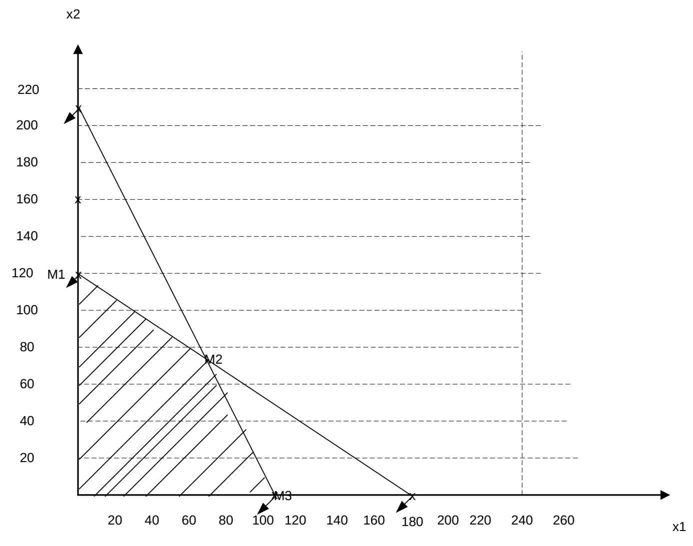
$x_1=0$ $x_2=210$

$x_1=105$ $x_2=0$

$2x_1 + 3x_2 \leq 360$

$x_1=0$ $x_2=120$

$x_1=180$ $x_2=0$



M1=(0,120)   Z1=5*0+6*120=720

M2=(70,75)   Z2= 5*70 +6*75=800

M3=(100,0)   Z3 =5*100+6*0= 500

MAX Z at x_1=70 , x_2=75 , z= 800

b) Min f(x) = $22x_1 + 25x_2$

Subject to the constraints:

$2x_1 + x_2 \geq 26$
$x_1 + x_2 \geq 14$
$x_1 \geq 0$ ، $x_2 \geq 0$

$2x_1 + x_2 \geq 26$

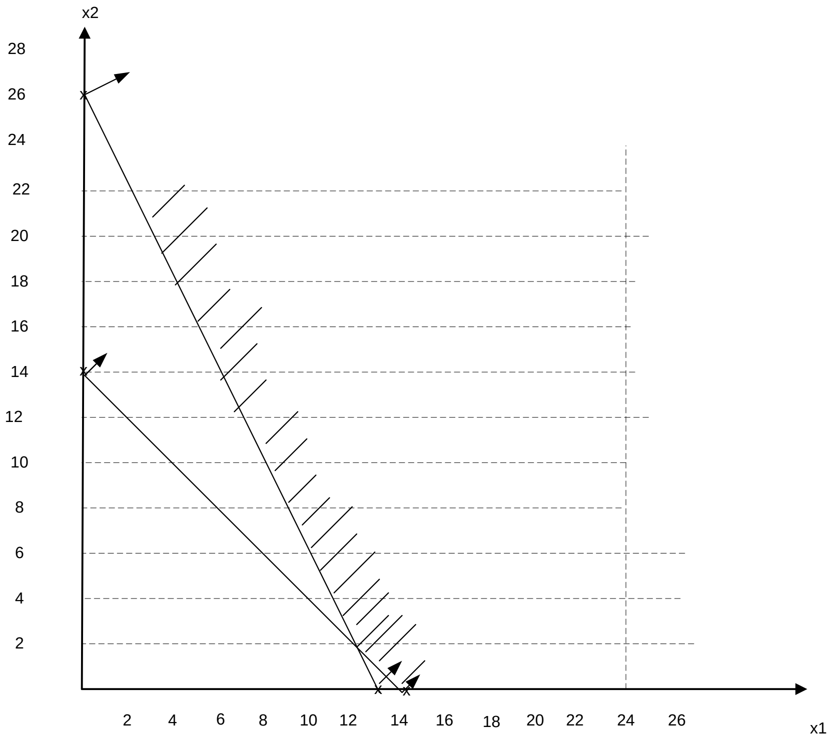
$x_1=0$ $x_2 =2$٦

$x_1=13$ $x_2 =0$

$x_1 + x_2 \geq 14$

$x_1=0$ $x_2 =14$

$x_1=14$ $x_2 =0$



M1=(0,26)   Z1=22*0+25*26=650

M2=(12,2)   Z2= 22*12+25*2=314

M3=(14,0)   Z3 =22*14+25*0= 308

MIN Z at $x_1=14$, $x_2=0$, z= 308

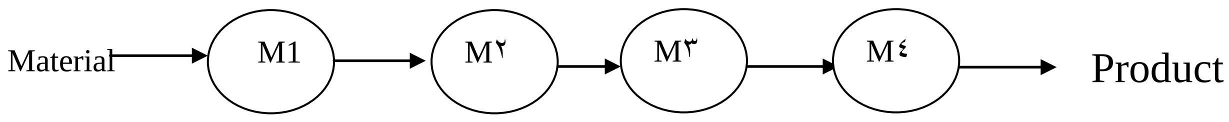
4. Name three advantages and three disadvantages for these types of layouts with drawing :

A. Product layout.

B. Process layout.

Solution (5)

A. Product layout.



Advantages: product layout provides the following benefits:

a) Low cost of material handling, due to straight and short route and absence of backtracking

b) Smooth and uninterrupted operations .

c) Continuous flow of work.

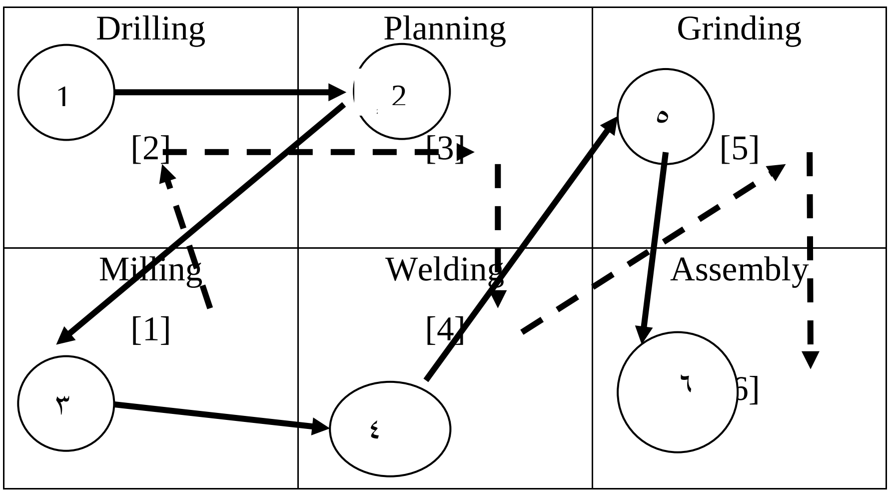
Disadvantages: Product layout suffers from following drawbacks:

a) Heavy overhead charges.

b) Breakdown of one machine will hamper the whole production process.

c) Lesser flexibility as specially laid out for particular product.

B. Process layout.



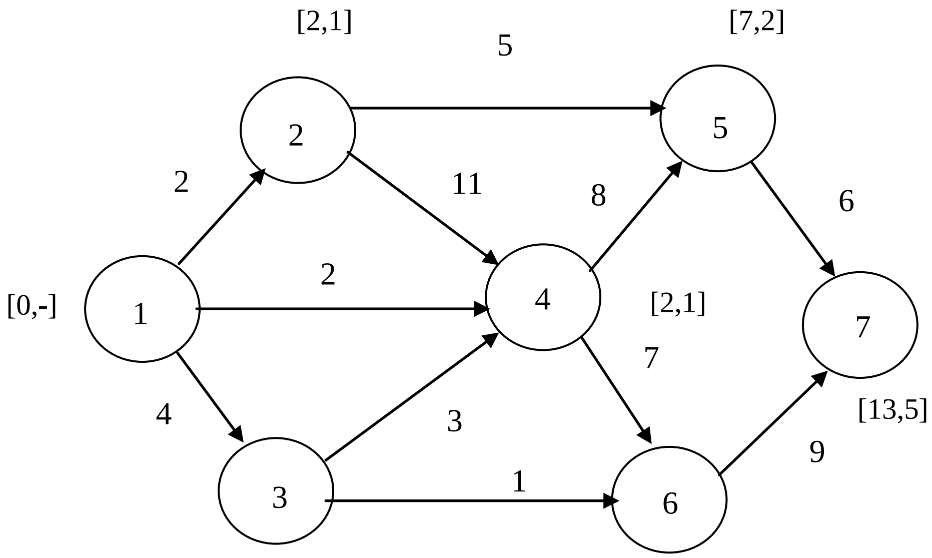
Advantages:

a) Lower initial capital investment in machines and equipments. There is high degree of machine utilization, as a machine is not blocked for a single product.
b) Breakdown of one machine does not result in complete work stoppage .
c) There is a greater flexibility of scope for expansion.

Disadvantages:

a. Material handling costs are high due to backtracking.
b. More skilled labor is required in higher cost.
c. Time gap or lag in production is higher.

C. **Compute the shortest path between node 1 and node 7 (and its length) in the network below**. For every link of the network, the length of that link is given in the picture



Node j	Computation of u_j	Label
1	$u_1 \equiv 0$	[0 ,-]
2	$u_2 = u_1+d_{12}$ = 0+2 =2, from 1	[2 ,1]
3	$u_3 = u_1+d_{13}$ = 0+4 =4, from 1	[4 ,1]
4	u_4 = min { u_1+d_{14} , u_2+d_{24}, u_3+d_{34}} = min { 0+2 , 2+11 , 4+3 } = 2 from 1	[2 ,1]
5	u_5 = min { u_2+d_{25}, u_4+d_{45}} = min { 2+5 , 2+8 } = 7, from 2	[7 ,2]
6	u_6 = min { u_3+d_{36}, u_4+d_{46}} = min { 4+1 , 2+7} = 5, from 3	[5 ,3]
7	u_7 = min { u_5+d_{57}, u_6+d_{67}} = min { 7+6 , 5+9} = 13, from 5	[13 ,5]

(7) ← (5) ← (2) ← (1)