Benha University
Benha High Institute of Technology
January 2011 -Fall semester
Examiner: Dr. Hesham El-Batsh
Date: 23 January 2011

Department: *Mechanical Engineering*
Time: 3 hours
2nd year Civil engineering
Subject: Fluid Mechanics and Machinery

Model Answer

Question (1) [12 marks]

a) Fluids for which the shearing stress is linearly related to the rate of shearing strain are called Newtonian fluids. Fluids for which the shearing stress is not linearly related to the rate of shearing strain are called non-Newtonian fluids.

Fluid which its density changes when the pressure changes is called compressible fluid while the fluid which its density do not change with pressure is called incompressible fluids. All liquid are considered incompressible while all gases are compressible fluids.

Viscous fluids are the fluids which have friction between fluid layers and therefore, the shear stress is non-zero and is calculated from the Newton law of viscosity. In the non-viscous fluids, the viscosity is considered zero and therefore, the shear stress between fluid layers is zero.

The absolute pressure is calculated above the absolute zero reference while the gage pressure is measured above the local atmospheric pressure reference.

b) The velocity distribution of water in a 0.1 m radius pipe is given by the expression:

$u = 20\left(1 - \frac{r^2}{100}\right)$ cm/s   Where r is the radius in cm units.

- Draw the velocity profile over the cross-section [2 marks]
- Obtain the shear stress on the pipewall and at the center of the pipe. [2 marks]
  The viscosity of the water is 0.015 Pa.s

$$\frac{du}{dr} = 20 \times \frac{2r}{100}$$
$$\tau = \mu \frac{du}{dr} = \mu \times \ 20 \times \frac{2r}{100}$$

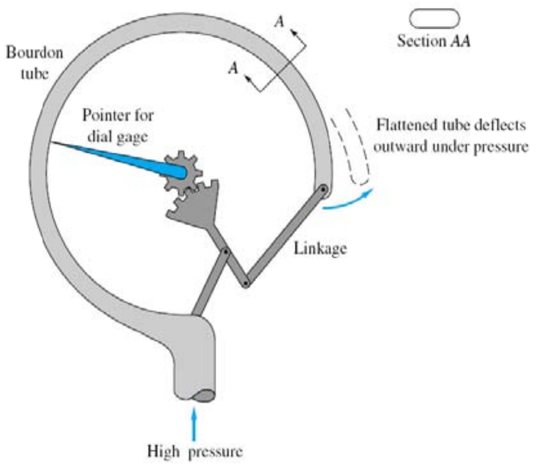
At the pipewall, r=0.1m, therefore

$$\tau = \mu \frac{du}{dr} = 0.015 \times \ 20 \times \frac{2 \times 10}{100} = 0.06 \ N/m^2$$

At the center of the pipe, r=0, therefore, $\tau = 0$

Question (2) [12 marks]

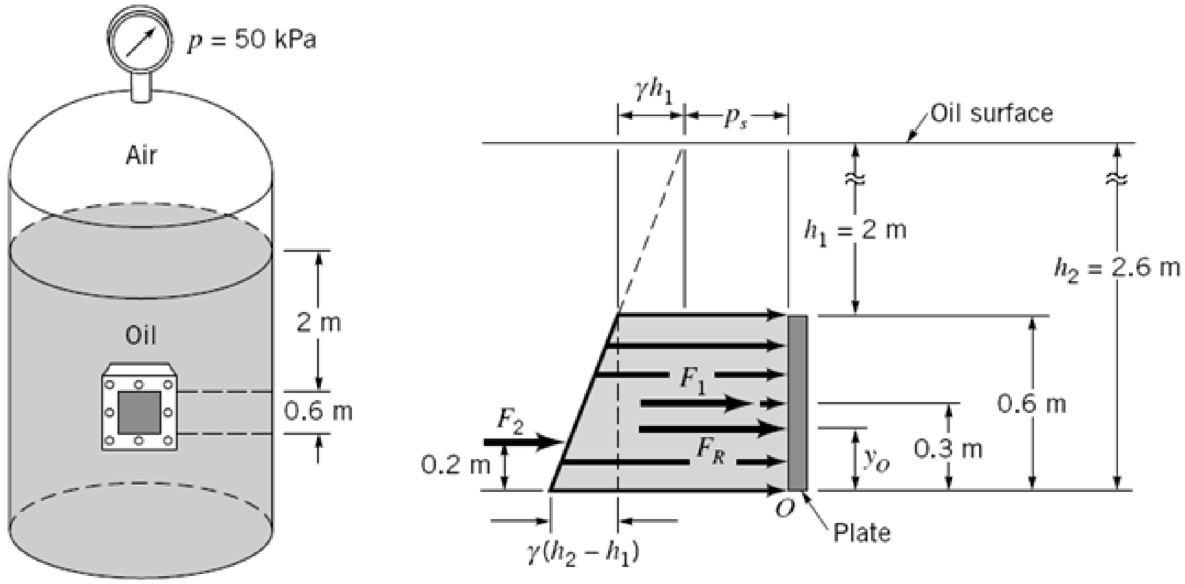
a) The idea of the mechanical pressure gage that when a pressure acts on an elastic structure the structure will deform, and this deformation can be related to the magnitude of the pressure.



b) Pressure of 0.4 m water = 0.4 x1000x9.81= 3924 Pa = 3.924 kN/m2= 29 mmHg
Vacuum pressure of 25 cm water = atmospheric pressure – 0.25x1000x9.81= 101396-2452 = 98944 N/m2= 740 mmHg

c)

The pressure distribution acting on the inside surface of the plate is shown in Fig. E2.8*b*. The pressure at a given point on the plate is due to the air pressure, p_s, at the oil surface, and



the pressure due to the oil, which varies linearly with depth as is shown in the figure. The resultant force on the plate (having an area A) is due to the components, F_1 and F_2, with

$$F_1 = (p_s + \gamma h_1)A$$
$$= [50 \times 10^3 \text{ N/m}^2 + (0.90)(9.81 \times 10^3 \text{ N/m}^3)(2 \text{ m})](0.36 \text{ m}^2)$$
$$= 24.4 \times 10^3 \text{ N}$$

and

$$F_2 = \gamma\left(\frac{h_2 - h_1}{2}\right)A$$
$$= (0.90)(9.81 \times 10^3 \text{ N/m}^3)\left(\frac{0.6 \text{ m}}{2}\right)(0.36 \text{ m}^2)$$
$$= 0.954 \times 10^3 \text{ N}$$

The magnitude of the resultant force, F_R, is therefore

$$F_R = F_1 + F_2 = 25.4 \times 10^3 \text{ N} = 25.4 \text{ kN} \qquad \textbf{(Ans)}$$

The vertical location of F_R can be obtained by summing moments around an axis through point O so that

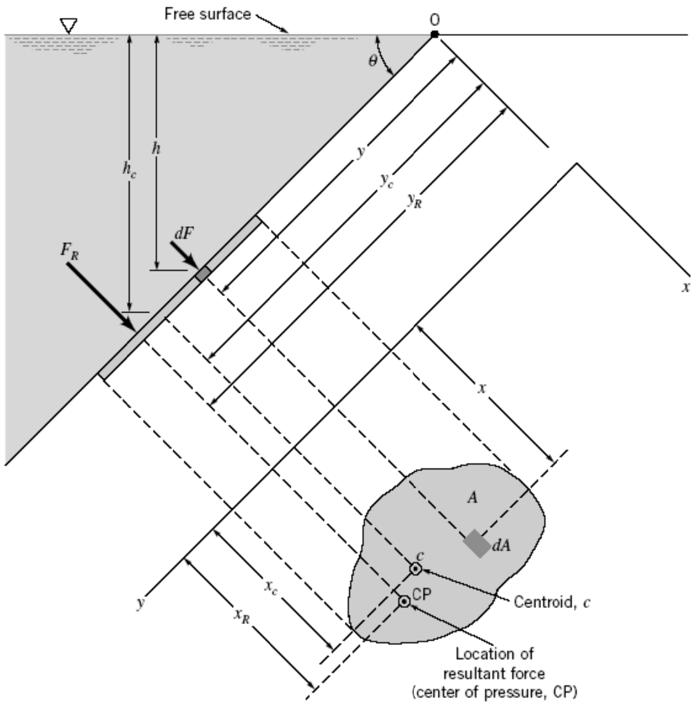
$$F_R y_O = F_1(0.3 \text{ m}) + F_2(0.2 \text{ m})$$

or

$$(25.4 \times 10^3 \text{ N})y_O = (24.4 \times 10^3 \text{ N})(0.3 \text{ m}) + (0.954 \times 10^3 \text{ N})(0.2 \text{ m})$$
$$y_O = 0.296 \text{ m} \qquad \textbf{(Ans)}$$

Question (3) [12 marks]

a)



$$F_R = \int_A \gamma h \, dA = \int_A \gamma y \sin\theta \, dA$$

where $h = y \sin\theta$. For constant γ and θ

$$F_R = \gamma \sin\theta \int_A y \, dA$$

$$\int_A y \, dA = y_c A$$

$$F_R = \gamma A y_c \sin\theta$$

More simply

$$\boxed{F_R = \gamma h_c A}$$

The moment of the resultant force must equal the moment of the distributed pressure force

$$F_R y_R = \int_A y \, dF = \int_A \gamma \sin\theta \, y^2 \, dA$$

Since, $F_R = \gamma A y_c \sin\theta$, therefore,

$$y_R = \frac{\int_A y^2 \, dA}{y_c A}$$

Thus, we can write

$$y_R = \frac{I_x}{y_c A}$$

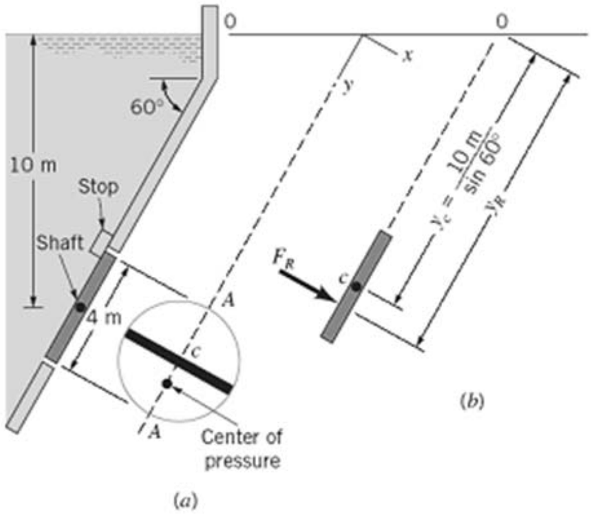
Using the parallel axis theorem to express I_x as

$$I_x = I_{xc} + Ay_c^2$$

Thus,

$$\boxed{y_R = \frac{I_{xc}}{y_c A} + y_c}$$

b)



To find the magnitude of the force of the water we can apply

$$F_R = \gamma h_c A$$

and since the vertical distance from the fluid surface to the centroid of the area is 10 m it follows that

$$F_R = (9.80 \times 10^3 \text{ N/m}^3)(10 \text{ m})(4\pi \text{ m}^2)$$
$$= 1230 \times 10^3 \text{ N} = 1.23 \text{ MN}$$

To locate the point (center of pressure) through which F_R acts, we use

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

$$I_{xc} = \frac{\pi R^4}{4}$$

$$y_R = \frac{(\pi/4)(2 \text{ m})^4}{(10 \text{ m}/\sin 60°)(4\pi \text{ m}^2)} + \frac{10 \text{ m}}{\sin 60°}$$
$$= 0.0866 \text{ m} + 11.55 \text{ m} = 11.6 \text{ m}$$

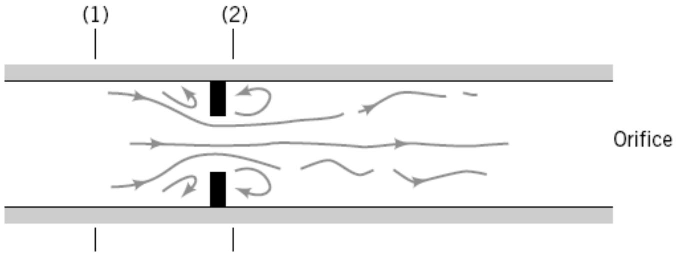
and the distance (along the gate) below the shaft to the center of pressure is

$$y_R - y_c = 0.0866 \text{ m}$$

Question (4) [12 marks]

a) The assumptions used to obtain Bernoulli equation are:
- The flow is steady
- The flow is incompressible
- The flow is non-viscous
- The flow is one-dimensional

b) An equation which can be used to obtain the flow rates using nozzles.



Between any two points, (1) and (2), on a streamline in steady, inviscid, incompressible flow the Bernoulli equation can be applied in the form

$$p_1 + \tfrac{1}{2}\rho V_1^2 + \gamma z_1 = p_2 + \tfrac{1}{2}\rho V_2^2 + \gamma z_2$$

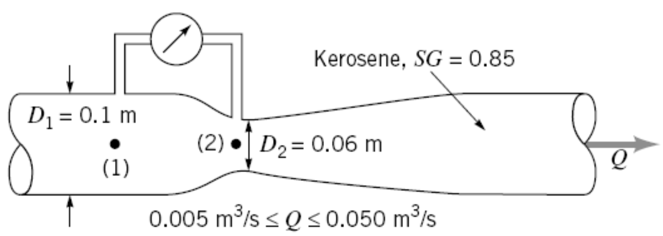
The continuity equation can be written as

$$Q = A_1 V_1 = A_2 V_2$$

Combining this equation with Bernoulli equation results in the following theoretical flowrate

$$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$$

c)



$$A_1 = 7.85 \times 10^{-3} m^2, A_2 = 2.83 \times 10^{-3} m^2$$

If $Q = 0.005\ m^3/s$

$$0.005 = 2.83 \times 10^{-3} \sqrt{\frac{2(p_1 - p_2)}{850\left[1 - \left(\frac{2.83}{7.85}\right)^2\right]}}$$

$p_1 - p_2$= 1154 Pa= 1.15 kPa

$$0.05 = 2.83 \times 10^{-3} \sqrt{\frac{2(p_1 - p_2)}{850\left[1 - \left(\frac{2.83}{7.85}\right)^2\right]}}$$

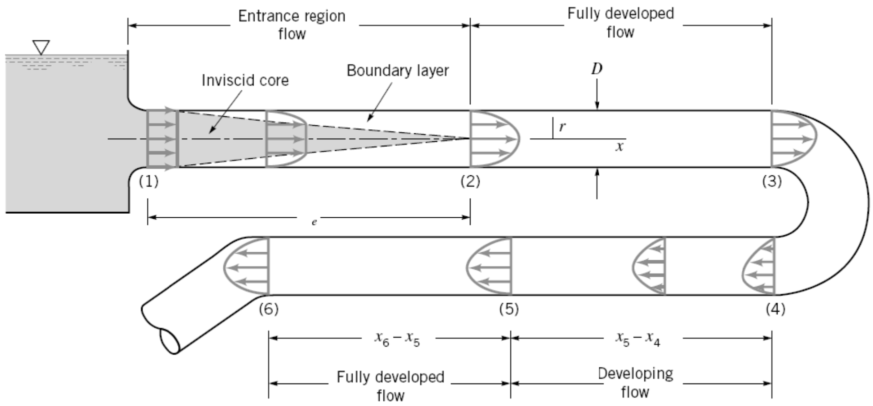
$p_1 - p_2$= 115418 Pa= 115 kPa

$$1.15 kPa \le p_1 - p_2 \le 115\ kPa$$

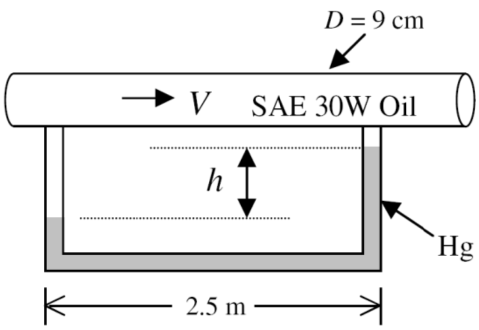
Question (5) [12 marks]

a)

- A steady flow is one in which the velocity may differ from point to point but do not change with time. The unsteady flow at any point in the fluid, the conditions change with time.
- In the uniform flow, the flow velocity is the same magnitude and direction at every point in the fluid. In the non-uniform the velocity is not the same at every point the flow.
- In the entrance region, the velocity profile changes from section to other section while for the fully developed flow the velocity profile has the same distribution at different cross-sections.



b)



$Re = \frac{\rho VD}{\mu} = \frac{891 \times 4.3 \times 0.09}{0.29} = 1189 < 2100$ The flow is laminar

$$Q = \frac{\pi}{4} D^2 v = \frac{\pi}{4} (0.09)^2 \times 4.3 = 0.027 m^3/s$$

Apply Bernoulli equation between points (1) and (2)

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_l$$

$$z_1 = z_2, v_1 = v_2$$

$$h_l = f \frac{L}{D} \frac{v^2}{2g}$$

$$f = \frac{64}{Re} = \frac{64}{1189} = 0.054$$

$$h_l = 0.054 \frac{2.5}{0.09} x \frac{(4.3)^2}{2x9.81} = 1.4\ m$$

$$P_1 - P_2 = \rho g h_l = 891 \times 9.81 \times 1.4 = 12237\ Pa$$

$$P_1 - P_2 = \rho g h_{Hg}$$

$$h_{Hg} = 0.092\ m$$