Benha University
Benha Higher Institute of Technology
Department of Mechanical Eng.
Subject: Fluid Mechanics Model Answer مادة ميكانيا الموائع كود م٢٠١
of the Final Exam Date: Jan./23/2011
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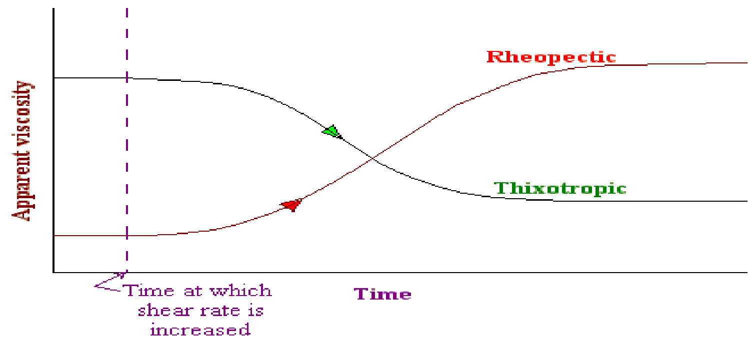
1-a) i) ***Thixotropic*** *fluids*: for which the dynamic viscosity decreases with the time for which shearing forces are applied. e.g. thixotropic jelly paints.

Rheopectic *fluids*: Dynamic viscosity increases with the time for which shearing forces are applied. e.g. gypsum suspension in water.



Effect of sudden change of shear rate on apparent viscosity of time-dependent fluids

ii) **Surface tension** is a phenomena that makes the interface between two fluids or between liquid and water behaves as elastic membrane.

iii) **Streakline** is a line passing through fluid particles originated from a given point (have earlier passed through a certain point in the flow field, **Timelines** are lines connecting specified particles at different time instants.

iv) ***Visco-elastic fluids*****:** Some fluids have elastic properties, which allow them to spring back when a shear force is released. e.g. egg white.

v/) **Potential flow:** Flow whose vorticity equal zero or Curl **V** = 0

vi) Fully developed fluid: at certain distance from the entrance of the pipe, when the boundary layer reaces the center of the pipe the flow is called fully developed.

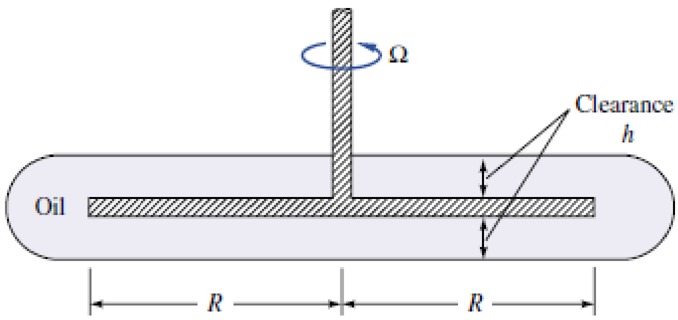
vii) **Boundary layer** : narrow region adjacent to the surface where the effect of the solid surface appears.

viii) **Entry length**: distance from the entrance of the pipe to the first section at which the flow becomes fully developed.

b) We find first the viscous torque (T) required:

$$dT = \tau dA.r = \mu \frac{du}{dy} 2\pi r dr.r = 2\pi\mu \frac{\Omega r}{h} r^2 dr$$

$$T = \frac{2\pi\mu\Omega}{h} \int_0^R r^3 dr = \frac{\pi\mu\Omega R^4}{2h}$$



$T = 0.1822 N.m$

$power = T.\Omega = 1.4577 watt$

c) Capillary rise or depression is given by the equation $h = 4\,\boldsymbol{\sigma}\cos(\boldsymbol{\theta})/(\boldsymbol{\rho}\, gd)$

h = - 3.935 mm depression

2-a)

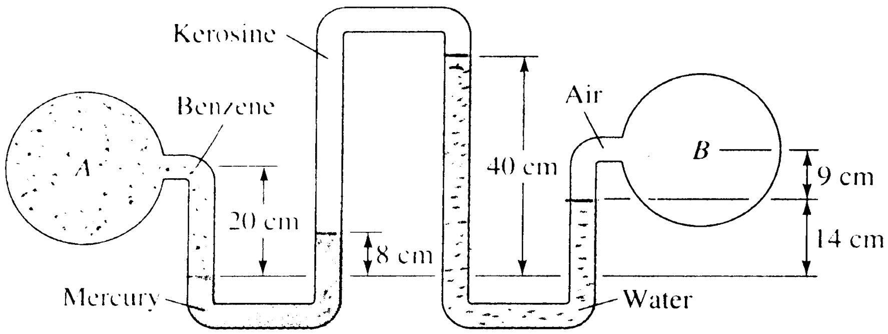


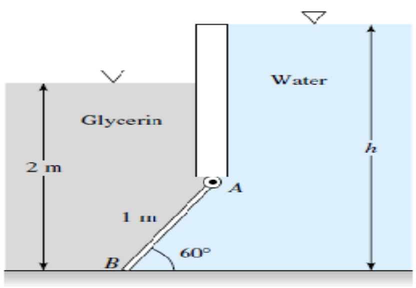
Figure 2

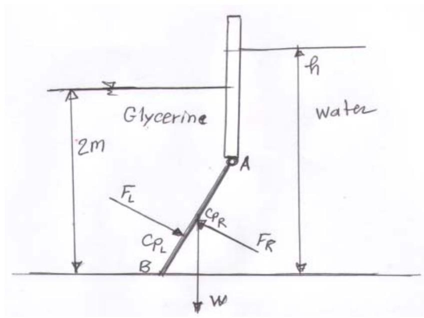
From figure 2 above we have:

$$P_A = 14.5\,psi = 1bar = 10^5\,Pa$$

$$P_A + \rho_{Benzene}g(0.2) - \rho_{Hg}g(0.08) - \rho_{kerosene}g(0.32) + \rho_w g(0.26) = P_B$$
$$\therefore P_B = 10^5 + 881\times9.81\times0.2 - 13600\times9.81\times0.08 - 0.804x9810\times0.32 + 9810x0.26$$
$$\therefore P_B = 91082Pa = 0.91082bar$$
$$\therefore P_B = 13.20688\,psi$$

b)





The left hand force acting by Glycerin

$F_L = \rho_G g h_{cG} A = 1264 \times 9.81 \times (2 - 0.5\sin60) \times 1 \times 1.2$

$= 23.3165$ KN

$Y_{cGL} = 1.8094$ m

$Y_{cpL} = Y_{cGL} + \frac{(1.2)(1)}{12 \times 1.8049 \times 1.2} = 1.85557$ m

The Distance between cp_L and A

$= 0{,}54617\ m$

The Right-hand force exerts by water

$F_R = 9810 \times (h - 0{,}5\sin 60) \times 1.2\ N$

$Y_{CPR} = Y_{CGR} + \dfrac{1.2 \times 1^3}{12 \times Y_{CGR} \times 1.2} = (1.1547h - 0{,}5) + \dfrac{1.2 \times 1^3}{12 \times (1.1547h - 0{,}5) \times 1.2}$

The Distence Between C_{PR} and A

$= 0.5 + \dfrac{1}{12 \times (1.1547h - 0{,}5)}$

Taking The moment about A ~~eq~~

$23.3165 \times 10^3 \times 0{,}54617 + 180 \times 9.81 \times 0{,}5\cos 60 = 9810 \times (h - 0{,}5\sin 60) \times 1.2 \times \left[0{,}5 + \dfrac{1}{12(1.1547h - 0{,}5)}\right]$

$13176.2 = 5886h - 2548.7 + \dfrac{981(h - 0{,}5\sin 60)}{1.1547h - 0{,}5}$

$15724.92 = 5886h + \dfrac{981(h - 0{,}433)}{1.1547h - 0{,}5}$

$18157.565h - 7862.46 = 6796.56h^2 - 2943h + 981h - 424.77$

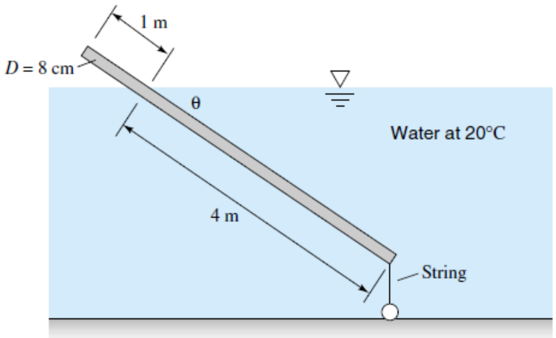
$\therefore 6796.56h^2 - 20119.565h + 7437.69 = 0$

$h^2 - 2.960257h + 1.09433 = 0$

$h_{1,2} = \dfrac{2.960257 \pm \sqrt{(2.960257)^2 - 4(1.09433)}}{2} = 2.52724\ m$ or $0{,}433\ m$

$h = 2.52724\ m$

2-c)



The weight of the rod is acting at the middle of the rod (C.G).

The up thrust R is acting upward at the middle of the immersed portion of the rod. Assume the tension T in the string.

From the equilibrium of forces R = W_{rod} + T $\rightarrow W_{rod} = \rho_{rod} V_{rod}\, g, R = \rho_{water} x 0.8 V_{rod}\, g$. Assume that the rod is making an angle θ with the horizontal and taking the moment about the lower end of the rod we get

$$\rightarrow W_{rod} x 2.5\cos\theta = R x 2\cos\theta \rightarrow \rightarrow 2.5\rho_{rod} V_{rod}\, g = 2\rho_{water} x 0.8 V_{rod}\, g \rightarrow \frac{\rho_{rod}}{\rho_{water}} = 0.64$$

The tension in the string T = R- Word = $\dfrac{\pi}{4}(0.08)^2 x 4 x 1000 x 9.81 - \dfrac{\pi}{4}(0.08)^2 x 5 x 640 x 9.81 = 39.448 N$

3-a) i-

$$\vec{V} = 10x^2 i - 20xyj + 100tk$$
$$\vec{V} = ui + vj + wk \rightarrow\rightarrow u = 10x^2, v = -20xy, w = 100tk$$

At the point (1,2,5) and time t=0.1sec, the velocity is;

$$\vec{V} = 10i - 40j + 10k$$
the acceleration $\vec{a} = a_x i + a_y j + a_z k$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \rightarrow\rightarrow a_x = 0 + (10x^2)(20x) + -20xy(0) + 100t(0) = 200x^3$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \rightarrow\rightarrow a_y = 0 + (10x^2)(-20y) + -20xy(-20x) + 100t(0) =$$
$$-200x^2y + 400x^2y = +200yx^2$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \rightarrow\rightarrow a_z = 100 + (10x^2)(0) + -20xy(0) + 100t(0) = 100$$

At the point (1,2,5) and time t=0.1sec, the acceleration is;

$$a_x = 200, a_y = 400, a_z = 100 \rightarrow \vec{a} = 200i + 400j + 100k$$

At time t=0

$$\vec{V} = ui + vj + wk \rightarrow\rightarrow u = 10x^2, v = -20xy, w = 0$$
$$\frac{dy}{dx} = \frac{v}{u} = \frac{10x^2}{-20xy} = \frac{x}{2y}$$
$$2ydy + xdx = 0 \rightarrow y^2 + \frac{x^2}{2} = Constant = 4.5$$

iii) The vorticity is given by : $\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -20y - 0 = -40S^{-1}$

3-b)i)

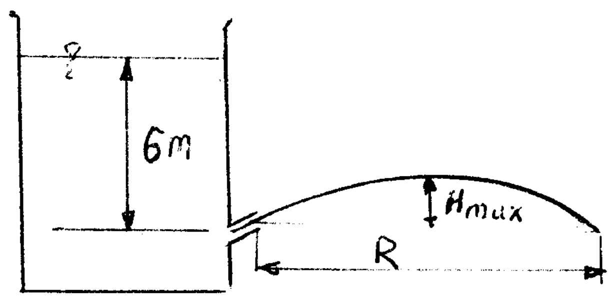


Figure 4

The maximum height H_{max} is given from the relation;

$H_{max} = H \sin^2\theta = 6.0x0.25 = 1.5$ m;

The range R is given by the equation: $R = 2H\sin 2\theta \rightarrow R = 2\times 6\times \sin 60^\circ$

$$R = 6\sqrt{3} = 10.392\text{m}$$

ii) The time to empty the tank is calculated as follows:

$$-A_T \frac{dh}{dt} = C_d A_o \sqrt{2gh} \rightarrow dt = -\frac{A_T}{C_d A_o \sqrt{2g}} \frac{dh}{\sqrt{h}}$$

$$t = -\frac{A_T}{C_d A_o \sqrt{2g}} \int_6^6 \frac{dh}{\sqrt{h}} = -\frac{2A_T\sqrt{h}}{C_d A_o \sqrt{2g}} \updownarrow_6^0 \rightarrow t = 737.34\text{sec}$$

3-c

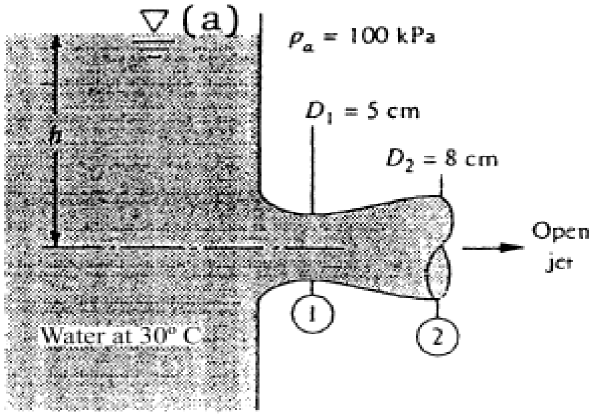


Figure 5

Applying Bernoulli equation between point a and the exit section 2

$V_2 = \sqrt{2gh}$; and from the continuity equation $V_1 = \left(\frac{D_2^2}{D_1^2}\right) V_2 = \frac{64}{25}\sqrt{2gh}$

And applying Bernoulli equation between point 1 and 2

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1$$

$$\frac{101325}{9810} + h + 0 = \frac{2340}{9810} + 7.1111111h + 0$$

$$h = 1.65113m$$

4-a)

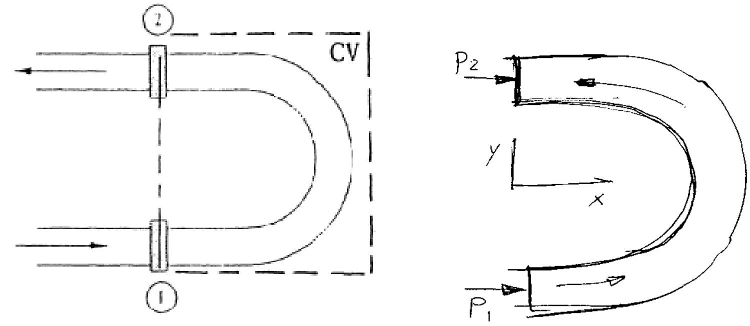


Figure 6

$$\sum F_x = \dot{m} u_{out} - \dot{m} u_{in}$$

$$U = \frac{Q}{A} = 11.7138 \ m/sec$$

$$(P_1 + P_2)A + R_x = 23 \cdot [-11.7138 - 11.7138]$$

$$(165 + 134) \times 10^3 \times 1.963495 \times 10^{-3} - R_x = -538.8348$$

$$R_x = 538.8348 + 587.085 = 1125.92 \ N$$

$$\sum F_y = \dot{m} V_{out} - \dot{m} V_{in}$$

$$-m_{water} g + R_y = 0 \Rightarrow R_y = \rho g A L = 14.446 N$$

$$R = \sqrt{R_x^2 + R_y^2} = 1126 \ N \qquad \Phi = \tan^{-1} \frac{R_y}{R_x} = 0.0764^\circ$$

b)

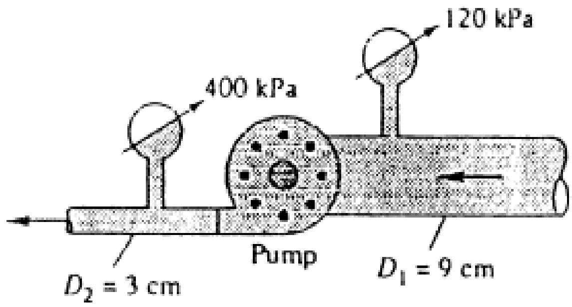


Figure 7

Applying Bernoulli equation between 1 and 2

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_p = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$h_p = \frac{p_2 - p_1}{\rho g} + \frac{V_2^2 - V_1^2}{2g}$$

The changes in z is negligible; i.e $z_1 = z_2$

$$V_1 = \frac{Q}{A_1} = 2.4888 m/\sec,$$

$$V_2 = \frac{Q}{A_2} = 22.4 m/\sec$$

Assume the x-direction is normal to the plate and opposite to F, and the y-direction is parallel to the plate and in the direction of Q_1.

There is no force acting on the plate in the y-direction; the only force is normal to the plate as shown in the above figure.

$V_1 = \frac{Q}{A_1} = 2.4888 m/\sec,$

$V_2 = \frac{Q}{A_2} = 22.4 m/\sec$

$h_p = 28.5423 + 25.25725 = 53.8 m$

Pump power = 8356.4 Watt
= 11.356 HP

5-a)

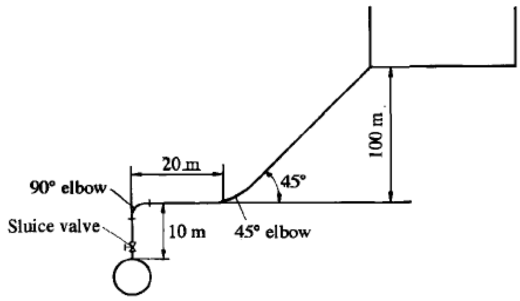


Figure 7

The speed of flow inside the pipe is given by $v = \frac{Q}{A} = \frac{\frac{1}{60}}{\pi \frac{0.1^2}{4}} = 2.122 m/\sec$

The total length of the pipe is L = 10 + 20 + 100$\sqrt{2}$ = 171.14 m

The losses at the pipe outlet is $\frac{v^2}{2g}$

Assume z_1 at pump outlet is 0, so z_2 is 110 m

Applying the energy equation $h_p = z_2 - z_1 + h_f + h_m$, where h_f is the friction losses given by:

$h_f = \frac{fL}{D}\frac{v^2}{2g}$ and h_m is the local losses given by: $h_m = (k_v + k_{90} + k_{45} + 1) \times \frac{v^2}{2g}$

$h_f = \frac{fL}{D}\frac{v^2}{2g} = 9.82$ m, and $h_m = (k_v + k_{90} + k_{45} + 1) \times \frac{v^2}{2g} = 0.6334 m$

$h_p = z_2 - z_1 + h_f + h_m = 110 + 9.82 + 0.6334 = 120.453 m$

Pump horse power = 19.694 kW

Brake horse power = 24.61767 kW

b) We have m = 7 physical parameters : deflection ,δ , wire length l, speed of fluid ,v , diameter of wire ,*d*, fluid density ,ρ , fluid viscosity ,μ, and modulus of elasticity ,E.

We have n=3 (three basic dimensions L, M, and T).

Dimensions of the physical parameters:

$\delta : [L]$,
$l : [L]$,
$v : [L T^{-1}]$,
$\rho : [ML^{-3}]$
$\mu : [ML^{-1}T^{-1}]$,
$d : [L]$,
$E : [ML^{-1}T^{-2}]$

We have m-n=3 π groups $\pi_1, \pi_2, \pi_3 \pi_4$,i.e $\Phi(\pi_1, \pi_2, \pi_3, \pi_4) = 0$

Choose ρ, l, v as repeating variables ,

$(\pi_1 = \delta\rho^{a_1} l^{b_1} v^{c_1}, \pi_2 = d\rho^{a_2} l^{b_2} v^{c_2}, \pi_3 = \mu\rho^{a_3} l^{b_3} v^{c_3}, \pi_4 = E\rho^{a_4} l^{b_4} v^{c_4})$

-For $\pi_1 = \delta\rho^{a_1} l^{b_1} v^{c_1} \rightarrow M^0L^0T^0 = [L][ML^{-3}]^{a_1} [L]^{b_1} [LT^{-1}]^{c_1}$

For M $\rightarrow 0 = a_1 \rightarrow a_1 = 0$

For L $\rightarrow 0 = 1 - 3a_1 + b_1 + c_1 = 1 + b_1 + c_1$

For T $\rightarrow 0 = 0 - c_1 \rightarrow c_1 = 0 \rightarrow \pi_1 = \dfrac{\delta}{1}$

-For $\pi_2 = Q\rho^{a_2} N^{b_2} D^{c_2} \rightarrow M^0L^0T^0 = [L^3T^{-1}][ML^{-3}]^{a_2} [T^{-1}]^{b_2} [L]^{c_2}$

For M $\rightarrow 0 = 0 + a_2 \rightarrow a_2 = 0$

For L $\rightarrow 0 = 3 - 3a_2 + c_2 \rightarrow c_2 = -3$

For T $\rightarrow 0 = -1 - b_1 \rightarrow b_1 = -1 \rightarrow \pi_2 = \dfrac{Q}{D^3N}$.

-For $\pi_3 = gh\rho^{a_3} N^{b_3} D^{c_3} \rightarrow M^0L^0T^0 = [L^2T^{-2}][ML^{-3}]^{a_3} [T^{-1}]^{b_3}$

For M $\rightarrow 0 = 0 + a_3 \rightarrow a_3 = 0$

For L $\rightarrow 0 = 2 - 3a_3 + c_3 \rightarrow c_3 = -2$

For T $\rightarrow 0 = -2 - b_1 \rightarrow b_1 = -2 \rightarrow \pi_3 = \dfrac{gh}{N^2D^2}$ so we have three non-dimensional grouping:

$\pi_1 = \dfrac{P}{\rho N^3 D^5}$, $\pi_2 = \dfrac{Q}{D^3N}$, and $\pi_3 = \dfrac{gh}{N^2D^2}$