INTERACTION OF SURFACE RADIATION WITH NATURAL CONVECTION AIR COOLING OF DISCRETE HEATERS IN A VERTICAL RECTANGULAR ENCLOSURE

R.Y. Sakr¹ and T.A. Ahmed²
¹ Mech. Eng. Dept., Shoubra Faculty of Engineering, Benha University, Egypt
² Mech. Power Eng. Dept., Faculty of Engineering, Cairo University, Egypt

ABSTRACT
This paper presents a numerical study that concerning the effect of surface radiation on natural convection cooling of four discrete isoflux heaters flush-mounted on a vertical wall of an air filled rectangular enclosure. The opposite side is isothermally cooled while the top and bottom sides as well as the spacing between the heaters are kept adiabatic. Two-dimensional mathematical model was developed based on solving the partial differential equations for the conservation of mass, momentum, and energy. A laminar flow is to be considered in the present model. Galerkin finite element method was utilized for the model formulation.

A numerical simulation is carried out to examine the parametric effects of enclosure aspect ratio, Rayleigh number, heater emissivity, adiabatic and cold surfaces emissivities, and the radiation parameter on the role played by the surface radiation in heat dissipation from the discrete heaters in the enclosure. The computed model results were verified through the comparison with those experimental results available in the literature and good agreement between them were noticed.

Several correlation equations in terms of aspect ratio, Rayleigh number, heater surface emissivity and the radiation parameter for the average Nusselt number for heaters, its radiative and convective components as well as the maximum surface temperature at the discrete heaters are obtained.

© 2007 Faculty of Engineering, Al-Azhar University, Cairo, Egypt. All rights reserved.

KEYWORDS: Natural Convection; Surface Radiation

1. INTRODUCTION
In the recent years, much attention is being paid to many practical applications involving heat transfer in vertical enclosures. Electronic equipment cooling, efficient building heating and design of solar collectors are some examples of these applications. Natural
convection cooling techniques have distinct advantages because of their low cost, easy maintenance and the absence of electromagnetic interface and operating noise. Although in most cases the flow is three dimensional, two-dimensional results are satisfactory, especially when considering the large reduction in computational effort. Comprehensive reviews of natural convection electronic cooling are available [1,2]. Under certain circumstances, such as operation in dusty or hazardous environment, electronic components are packaged within sealed enclosures. The components may be mounted to one vertical wall of the enclosure, while the other walls are cooled. This geometry is one of the fundamental configurations encountered in thermal management of electronic equipment and has received extensive attention since the pioneering study of Chu et al. [3] as indicated by the references cited in the recent works [4-6]. In most cases, there is an interaction of surface radiation and natural convection in many analyses. This interaction is omitted due to low emissivities, then radiation contribution has been taken care of by adding a correction using simple formulae like that between infinite parallel plates facing each other, Hoogendoorn [7].

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>element area</td>
</tr>
<tr>
<td>AR</td>
<td>aspect ratio H/W</td>
</tr>
<tr>
<td>dn</td>
<td>distance between two adjacent nodes in the normal direction to the wall boundary</td>
</tr>
<tr>
<td>E</td>
<td>total number of elements</td>
</tr>
<tr>
<td>Fij</td>
<td>shape factor between surfaces i, j</td>
</tr>
<tr>
<td>[F]</td>
<td>load vector, Eq. (16).</td>
</tr>
<tr>
<td>g</td>
<td>gravity vector</td>
</tr>
<tr>
<td>G</td>
<td>bounded domain</td>
</tr>
<tr>
<td>h</td>
<td>heat transfer coefficient, W/m²K</td>
</tr>
<tr>
<td>H</td>
<td>enclosure height</td>
</tr>
<tr>
<td>J</td>
<td>dimensionless radiosity</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity, W/mK</td>
</tr>
<tr>
<td>[K]</td>
<td>stiffness matrix</td>
</tr>
<tr>
<td>l</td>
<td>heater length, m</td>
</tr>
<tr>
<td>L</td>
<td>dimensionless heater length, l/W</td>
</tr>
<tr>
<td>N1,2,3</td>
<td>interpolation functions</td>
</tr>
<tr>
<td>NNS</td>
<td>total number of radiation surfaces</td>
</tr>
<tr>
<td>Nr</td>
<td>radiation parameter, σT⁴ / qₜ</td>
</tr>
<tr>
<td>Nu</td>
<td>convection Nusselt number</td>
</tr>
<tr>
<td>NuR</td>
<td>radiation Nusselt number</td>
</tr>
<tr>
<td>Nuₜ</td>
<td>total Nusselt number</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number, vα</td>
</tr>
<tr>
<td>q''</td>
<td>heat flux, W/m²</td>
</tr>
<tr>
<td>qₜ</td>
<td>heat flux of heater, W/m²</td>
</tr>
<tr>
<td>q</td>
<td>dimensionless heat flux,</td>
</tr>
<tr>
<td>Ra</td>
<td>modified Rayleigh number, g β qₜ W²/(kνα)</td>
</tr>
<tr>
<td>s</td>
<td>spacing between heaters, m</td>
</tr>
<tr>
<td>S</td>
<td>dimensionless spacing between heaters, s/W</td>
</tr>
<tr>
<td>T</td>
<td>temperature, K</td>
</tr>
<tr>
<td>TR</td>
<td>temperature ratio, T/Tc</td>
</tr>
<tr>
<td>u</td>
<td>velocity component in x-direction</td>
</tr>
<tr>
<td>v</td>
<td>velocity component in y-direction</td>
</tr>
<tr>
<td>W</td>
<td>enclosure width, m</td>
</tr>
<tr>
<td>x, y</td>
<td>Cartesian coordinates</td>
</tr>
<tr>
<td>X, Y</td>
<td>dimensionless Cartesian coordinates x/W, y/W</td>
</tr>
</tbody>
</table>

Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Thermal diffusivity of the fluid, m²/s</td>
</tr>
<tr>
<td>β</td>
<td>thermal expansion coefficient, 1/K</td>
</tr>
<tr>
<td>Γ</td>
<td>domain boundary</td>
</tr>
<tr>
<td>θ</td>
<td>dimensionless temperature</td>
</tr>
<tr>
<td>ν</td>
<td>Viscosity, kg/m.s</td>
</tr>
<tr>
<td>ρ</td>
<td>Density, kg/m³</td>
</tr>
<tr>
<td>ψ</td>
<td>stream function, m²/s</td>
</tr>
<tr>
<td>Ψ</td>
<td>dimensionless stream function, ψ/α</td>
</tr>
<tr>
<td>ε</td>
<td>surface emissivity</td>
</tr>
<tr>
<td>σ</td>
<td>Stefan-Boltzmann constant, 5.667x10⁻⁸ W/m²K⁴</td>
</tr>
<tr>
<td>ω</td>
<td>vorticity, 1/s</td>
</tr>
<tr>
<td>Ω</td>
<td>dimensionless vorticity, ωW²/α</td>
</tr>
<tr>
<td>ΔN</td>
<td>dimensionless distance between two adjacent nodes in the normal direction to the wall boundary.</td>
</tr>
</tbody>
</table>

Superscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>element level</td>
</tr>
<tr>
<td>T</td>
<td>Transpose</td>
</tr>
<tr>
<td>R</td>
<td>Radiation</td>
</tr>
</tbody>
</table>

Subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Adiabatic</td>
</tr>
<tr>
<td>c</td>
<td>Cold</td>
</tr>
<tr>
<td>h</td>
<td>hot, heater</td>
</tr>
<tr>
<td>R</td>
<td>radiation</td>
</tr>
<tr>
<td>w</td>
<td>wall</td>
</tr>
</tbody>
</table>
There are a few studies available in the literature concerning the effect of radiation exchange among surfaces of an enclosure bounding a non-participating fluid. The interaction of surface radiation with transient natural convection in an air-filled rectangular cavity surrounded by one-dimensional conducting walls was analyzed numerically by Larson and Viskanta [8]. Their work was further extended by Kim and Viskanta [9] by considering two-dimensional conduction in the cavity walls.

Balaji and Venkateshan [10] studied the effect of surface radiation on natural convection in a rectangular enclosure. Their model is based on Gosman's finite volume method. A careful review of the literature suggests that the work of Asako and Nakamura [11] is one of the earnest attempts to probe into the effect of surface radiation on natural convection in an enclosure.

For the interaction of surface radiation with natural convection in a discretely heated enclosure, there exists a very little previous work in the literature. Ho and Chang [12] presented basic insights of the role played by surface radiation in cooling of multiple discrete heaters enclosed in a vertical rectangular enclosure. They achieved their analysis numerically by using a finite difference method. In this paper, a parametric study to investigate the interaction of surface radiation with natural convection in a discretely heated enclosure having different aspect ratios using Galerkin finite element method is carried out.

2. MATHEMATICAL FORMULATION

The present problem is illustrated schematically in Fig. (1). Simultaneous heat dissipation by natural convection and surface radiation from four discrete flush-mounted heaters of iso-flux $q_h$ on the left vertical wall of an air filled rectangular enclosure. The enclosure width and height are $W$ and $H$ respectively. The discrete heaters of lengths, $l$, are located uniformly with spacing, $s$. The right vertical wall of the enclosure is isothermally kept at temperature $T_c$; and the horizontal walls as well as the left vertical regions between the discrete heaters are assumed adiabatic. The multi-mode heat transfer process within the enclosure is modeled as two-dimensional, steady and laminar natural convection with simultaneous radiation exchange between the end walls of the enclosure. The air is assumed to be radiative non-participating with constant properties and Boussinesq approximation is assumed to be valid. Viscous dissipation and compressibility effects are also neglected.

2.1 Governing Equations

The dimensionless governing equations for the problem can be expressed in terms of temperature, vorticity, and stream function as follows:

\[
\frac{\partial \Psi}{\partial y} \frac{\partial \Theta}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial \Theta}{\partial y} = \frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} \tag{1}
\]

\[
\frac{\partial \Psi}{\partial y} \frac{\partial \Omega}{\partial x} - \frac{\partial \Psi}{\partial x} \frac{\partial \Omega}{\partial y} = Ra Pr \frac{\partial \Theta}{\partial X} + Pr \left( \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right) \tag{2}
\]

\[
\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\Omega \tag{3}
\]

2.2 Boundary Conditions

1- The dimensionless temperature boundary conditions may be written as follows:

At $X=0$, $-\frac{\partial \Theta}{\partial X} + q^e = 1.0$ (at heaters), $-\frac{\partial \Theta}{\partial X} + q^e = 0.0$ (elsewhere) \hspace{1cm} (4a)
At X=1, Θ = 1.0  
At Y=0, 0≤X≤1.0 \( - \frac{\partial \Theta}{\partial X} + q^R = 0.0 \)  
At Y=AR, 0≤X≤1.0 \( - \frac{\partial \Theta}{\partial X} + q^R = 0.0 \)

Where; the dimensionless local radiative heat flux \( q^R = \frac{q^{R*}}{q_h} \) can be evaluated by solving the equations for radiative exchange between the end walls of the enclosure. The net radiative flux at any point on the enclosure surface \( i \) can be expressed as:

\[
q^R_i = \left( \frac{\varepsilon_i}{1-\varepsilon_i} \right) [N \Omega TR^4 - J_i]
\]

Where; \( TR = \frac{T}{T_c} \)  
\( NR = \frac{\sigma T_c}{q_h} \)

The dimensionless radiosity \( J_i \) for the \( i \)th surface element can be determined by solving the following equation:

\[
J_i = \varepsilon_i + (1 - \varepsilon_i) \sum_{j=1}^{NNS=84} F_{ij} J_j
\]

The shape factors \( F_{ij} \) between the two surfaces \( i, j \) in the above equation can be determined by different formulae according to the relative position between surfaces. In the present study, \( F_{ij} \) is calculated by using the crossed string method [13].

2- Stream function boundary conditions
At the solid wall boundaries the values of velocity components \( u \) and \( v \) are zero; so

\[
\frac{\partial \Psi}{\partial Y} = 0.0 \quad (7a)
\]

\[
\frac{\partial \Psi}{\partial X} = 0.0 \quad (7b)
\]

Equations (7a) and (7b) indicate that the magnitude of the stream function over the wall, \( \Psi_w \), is constant. Moreover, \( \Psi_w \) is usually assigned a value of zero on the solid boundaries so, \( \Psi_w = 0.0 \)

3- Vorticity boundary conditions
By using the definition of the vorticity and applying no-slip boundary condition the following equations can be written;

\[
\omega = - \frac{d^2 \Psi}{(dn)^2} \quad (9)
\]

Equation (9) can be approximated by using finite difference formulas and then expressed in terms of dimensionless variables as follows:

\[
\Omega_w = \frac{2(\Psi_{w+1} - \Psi_w)}{(\Delta N)^2} \quad (10)
\]

where;

\( w \) refers to the nodal value at the no slip wall.
w+1 refers to the adjacent interior node.
ΔN is the dimensionless distance separating this node pair.

Equations (1), (2) and (3) are solved with the relevant boundary conditions given by Eqs. (4), (8) and (10) to determine the temperature, vorticity and stream function distributions.

2.3 Heat Transfer
The local Nusselt number at the discretely heated surfaces is defined by [14]:

\[ Nu_h = \frac{1}{\Theta_h} \]  

and the average Nusselt number is given by:

\[ \bar{Nu} = \frac{1}{H_h} \int Nu_h dL \]  

This average Nusselt number is due to the contribution of both natural convection and surface radiation heat transfer modes. So, the average Nusselt number due to radiation and convection can be written as:

\[ Nu_R = \left( \frac{q_R}{q} \right) \bar{Nu} \]  

\[ Nu_c = \left( \frac{q_c}{q} \right) \bar{Nu} \]  

3. NUMERICAL SOLUTION AND FINITE ELEMENT FORMULATION
The system of equations (1), (2) and (3) forms a set of quasilinear elliptic equations. So, the solutions of the system of equations for \( \Psi \), \( \Omega \) and \( \Theta \) will be continuous in the domain G. Hence, the system of equations was solved through an iterative procedure. Initially the stream function is assumed to have a zero value everywhere and Eq. (1) is then solved as a linear equation for \( \Theta \). This solution describes the temperature distribution for the pure conduction case. This temperature distribution and the associated stream function field are then substituted into Eq. (2) from which \( \Omega \) is obtained. Finally the obtained vorticity distribution is used in Eq. (3) and an improved value of \( \Psi \) is obtained. The cycle of iteration is repeated until the values of \( \Psi \) for two consecutive calculations are within \( 10^{-4} \).

The system of equations (1-3) is solved using the Galerkin based finite element method [15, 16]. The objective of the finite element is to reduce the system of governing equations into a discretized set of algebraic equations. The procedure begins with the division of the continuum region G, of interest into a number of simply shaped regions called finite elements.

### The Finite Element Formulation
The temperature, vorticity and the stream function in any element (triangle) of the discretized domain as shown in Fig. (1b) can be represented in terms of nodal temperature, vorticity and stream function respectively by the following simple polynomials:

\[ \Theta^e = \sum_{m=1}^{3} N_m \Theta_m \]  

\[ \Omega^e = \sum_{m=1}^{3} N_m \Omega_m \]
\[ \Psi^e = \sum_{m=1}^{3} N_m \Psi_m \]  

(14c)

where:

\[ N_1 = \frac{1}{2A} \ (a_1 + b_1 X + c_1 Y) \]  

(15a)

\[ N_2 = \frac{1}{2A} \ (a_2 + b_2 X + c_2 Y) \]  

(15b)

\[ N_3 = \frac{1}{2A} \ (a_3 + b_3 X + c_3 Y) \]  

(15c)

where;

A = area of the triangle 123

\[ a_1 = X_2 Y_3 - X_3 Y_2 \]

\[ b_1 = Y_2 - Y_3 \]

\[ c_1 = X_3 - X_2 \]

The interpolation functions \([N_1, N_2, N_3]\) in Eqs. (14) are derived from an assumption of linear variation of temperature, vorticity and stream function in the element. The approximate expressions of the system variables are substituted into the governing equations (1)-(3) and the global errors are minimized using the above interpolation functions \(N_i \) \( (i = 1, 2, 3) \) as weighting functions. The solution of Eqs. (1), (2) and (3) that satisfies the boundary conditions given by Eqs. (4), (8) and (10), can be written after weighted integration over the domain G and the application of Green’s theorem, in the equivalent matrix form as:

\[
\begin{align*}
[K_1] \{\Theta\} &= \{F_1\} \quad \text{(16a)} \\
[K_2] \{\Omega\} &= \{F_2\} \quad \text{(16b)} \\
[K_3] \{\Psi\} &= \{F_3\} \quad \text{(16c)}
\end{align*}
\]

where,

\[
[K_1] = [K_{II}] + [K_{I I}]
\]

\[
[K_I] = \sum_{e=1}^{E} \int_{G_e} \left( \frac{\partial [N]^T}{\partial X} \frac{\partial [N]}{\partial X} + \frac{\partial [N]^T}{\partial Y} \frac{\partial [N]}{\partial Y} \right) \ dG^e
\]

\[
[K_{II}] = \sum_{e=1}^{E} \int_{G_e} \left( [\Psi_X \frac{\partial [N]}{\partial Y} - \Psi_Y \frac{\partial [N]}{\partial X}] \right) \ dG^e
\]

\[
\{F_1\} = \sum_{e=1}^{E} \int_{\Gamma_e} [N]^T \frac{\partial N}{\partial X} d\Gamma^e + \sum_{e=1}^{E} \int_{\Gamma_e} [N]^T \frac{\partial N}{\partial Y} d\Gamma^e
\]

where;

\[ E = \text{total number of elements, } G \text{ bounded domain, } \Gamma \text{ domain boundary,} \]

\[ \Psi_X = \frac{\partial \Psi}{\partial X}, \quad \Psi_Y = \frac{\partial \Psi}{\partial Y} \]

Similarly, \([K_2], [K_3], \{F_2\} \text{and } \{F_3\}\) can be written in the same manner. Equations (16a, 16b, 16c) give three sets of linear equations which have been solved by Gauss
elimination method. The finite element formulation and the resulting linear equations were solved through a computer program written here in FORTRAN code.

4. MODEL VALIDATION
To check the consistency and reliability of the present analysis, the same conditions employed by Ramesh and Venkateshan [17] were used in the predictions. Figure (2) shows the obtained results compared with the experimental data of Ramesh and Venkateshan. Considerable discrepancy is noticed in the case of low cold and hot surface emissivity operating conditions while good agreement is noticed for high emissivity of hot and cold surface emissivity.

5 RESULTS AND DISCUSSIONS
From the foregoing mathematical formulation, the dimensionless parameters that govern the present problem include the modified Rayleigh number $Ra$, the radiation parameter $NR$, and the surface emissivity of the discrete heaters, cold wall and the adiabatic surfaces $\varepsilon_h$, $\varepsilon_c$, $\varepsilon_a$ respectively. Furthermore, the problem is solved for different enclosures with different aspect ratios $AR$. So, numerical calculations have been carried out for rectangular enclosures with the previous parameters having the following ranges: $1 \leq AR \leq 10$, $10^3 \leq Ra \leq 5 \times 10^6$, $0.2 \leq \varepsilon_h \leq 0.8$, and $0.75 \leq NR \leq 2.5$. Moreover, the dimensionless heater size $l/H$ and spacing $s/H$ on the vertical left side are fixed at 0.033 and 0.173 respectively.

5.1 Flow and Temperature Fields
Figure (3) shows the computed steady-state streamlines and isotherms obtained with the aid of the present numerical scheme for vertical enclosure having four heaters flush mounted on the left side; for $AR=5$ and $Ra=10^5$. The figure illustrates two cases namely: “without” surface radiation and “with” surface radiation taking radiation parameter $NR=1.5$, the surface emissivities for heaters, cold and adiabatic surfaces are all equal and having a value of 0.5. The figure indicates that the surface radiation tends to decrease the values of stream functions and temperatures; this means a decrease of the heat transfer by natural convection rate but with an associated enhancement of the overall heat transfer rate. However, these variations are insignificant, which means that the radiation heat transfer at this Rayleigh number is relatively small. But for $Ra=5 \times 10^6$, Fig. (4) shows that, with the coupling of surface radiation, thermal boundary layer along the cold wall tends to degenerate considerably, thereby reducing convective heat transfer rate there. The same behavior can be nearly noticed in Fig. (5) for $AR=7$, with all other parameters kept unchanged.

5.2 Heat Transfer Results
The effect of Rayleigh number on the heat transfer process that takes place by either radiation or natural convection mode for each discrete heater is illustrated in Fig. (6a-d). The figure shows sharp increase of radiation Nusselt number and slight decrease of convection Nusselt number with the increase of Rayleigh number. Comparing with “without” radiation case, as Rayleigh number increases enhancement in total Nusselt number continues.

The variation of the maximum temperature for each heater with Rayleigh number is shown in Fig. (7), for $AR=5$, $NR=1.5$, and $\varepsilon_h=\varepsilon_c=\varepsilon_a=0.5$. From the figure, it can be noticed that, as Rayleigh number increases, the maximum temperature decreases and the first heater (on the top) has the greatest temperature in the enclosure. The effect of the enclosure aspect ratio on the average Nusselt number is illustrated for each heater at $Ra=10^6$, $NR=1.5$, $AR=5$, and $NR=2$. The results indicate that the average Nusselt number decreases with increasing aspect ratio.
and $\varepsilon_h=\varepsilon_c=\varepsilon_a=0.5$ in Fig. (8a-d). The figure shows that the effect of the aspect ratio on the radiation mode heat transfer is insignificant. Furthermore, it is noticed that large increase of convective Nusselt number with the increase of the enclosure aspect ratio. Compared with “without” radiation case, a slight increase of the total Nusselt number with the increase of the enclosure aspect ratio is noticed. The effect of aspect ratio on the maximum heater temperature is depicted in Fig. (9). As the aspect ratio increases, the maximum temperature decreases. Also, surface radiation heat transfer leads to the decrease of the maximum temperature on the surface of the heaters for all aspect ratios considered in the present study, see Fig. (9).

The effect of surface radiation, represented by the radiation parameter $N_R$, on the average Nusselt number for the four discrete heaters is illustrated in Fig. (10a-d). From the figure it is can be concluded that the radiation Nusselt number increases as the radiation parameter increases, while the convection Nusselt number decreases as the radiation parameter increases. Furthermore, the figure shows increase of the total Nusselt number with the increase of the radiation parameter. Comparing with “without” radiation case, the surface radiation presented in radiation parameter, enhances the process of heat transfer.

Figure (11) shows the variation of the maximum dimensionless temperature on the heaters surfaces with the radiation parameter. It is noticed from the figure, that the maximum heaters surfaces temperature decreases markedly with the increase of the radiation parameter and this is due to heat transfer enhancement.

The effect of surface radiation on the heat transfer process is investigated by studying the effect of emissivity of the heaters surfaces on the average Nusselt number for each heater in Fig.(12a-d) for $Ra=10^6$, $N_R=1.5$, $\varepsilon_h=\varepsilon_c=\varepsilon_a=0.5$. It can be seen from the figure that as the emissivity of the heaters surfaces increases the radiation Nusselt number increases as well. Whereas the convection Nusselt number decreases and the total Nusselt number increases compared with the case of “without” surface radiation condition. The effect of heater emissivity, $\varepsilon_h$, on the maximum temperature of heaters is depicted in Fig.(13). For heater emissivity up to 0.6 a slight decrease of surface temperature with the increase of heaters emissivity is noticed. When $\varepsilon_h$ increases above 0.6, a sharp decrease of the maximum temperature occurs. Finally, the effect of emissivity of both adiabatic and cold surfaces $\varepsilon_a$, $\varepsilon_c$ respectively on the radiative heat transfer is investigated. However, it is found that their effects are insignificant. For the sake of brevity, only the effect of adiabatic surface emissivity is illustrated in Table (2).

<table>
<thead>
<tr>
<th>$\varepsilon_a$</th>
<th>$\theta_{\text{max}}$</th>
<th>$N_{\text{th}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.13862</td>
<td>9.4677</td>
</tr>
<tr>
<td>0.3</td>
<td>0.13861</td>
<td>9.4654</td>
</tr>
<tr>
<td>0.4</td>
<td>0.13860</td>
<td>9.4657</td>
</tr>
<tr>
<td>0.5</td>
<td>0.13860</td>
<td>9.4695</td>
</tr>
<tr>
<td>0.6</td>
<td>0.13859</td>
<td>9.4660</td>
</tr>
<tr>
<td>0.7</td>
<td>0.13866</td>
<td>9.4665</td>
</tr>
<tr>
<td>0.8</td>
<td>0.13859</td>
<td>9.4666</td>
</tr>
<tr>
<td>0.9</td>
<td>0.13858</td>
<td>9.4672</td>
</tr>
</tbody>
</table>
The local temperature distribution on the left side (flush heater side) with and without radiation cases for different Rayleigh numbers is illustrated in Fig. (14). From the figure, it is depicted that the maximum temperature attained inside the enclosure is in the upper heater surface. Furthermore, the surface radiation decreases the temperature level due to the heat transfer enhancement especially at higher Rayleigh numbers.

5.3 Nusselt Number and Maximum Temperature Correlations

Based upon present numerical results, general correlations that accounts for the governing parameters were developed by expressing the average Nusselt number, its radiative and convective components on the heaters’ surfaces as well as the maximum temperature that attained inside the enclosure as:

\[
\overline{Nu} = 0.1796 AR^{0.5563} Ra^{0.2387} \epsilon_h^{0.1454} N_R^{0.1538}
\]

\[
Nu_R = 5.339 \times 10^{-5} AR^{0.3507} Ra^{0.0739} \epsilon_h^{1.868} N_R^{2.0333}
\]

\[
Nu_C = 0.3268 AR^{0.5725} Ra^{0.1603} \epsilon_h^{0.1858} N_R^{-0.1782}
\]

\[
\theta_{max} = 4.823 AR^{-0.497} Ra^{-0.214} \epsilon_h^{-0.1655} N_R^{-0.1668}
\]

with errors less than 9.5%

From the indices of the above correlation, it is clear that the relative powers reflect the influence of each parameter on the interaction between surface radiation and natural convection. While in the case of “without” surface radiation, the average Nusselt number and the maximum temperature inside the enclosure are given by:

\[
\overline{Nu} = 0.284 AR^{0.5944} Ra^{0.1848}
\]

with an error of 9.5%

\[
\theta_{max} = 2.3596 AR^{-0.5152} Ra^{-0.1057}
\]

with an error of 0.1%

6. CONCLUSIONS

The interaction of surface radiation with natural convection in an air filled enclosures was investigated numerically. From the findings in the previous item of this paper, the following conclusions are drawn:
1. The increase of Rayleigh number leads to an increase of radiation Nusselt number while it leads to an increase of the convection Nusselt number to some value then decrease in convection Nusslet number with further increase of Rayleigh number.
2. Surface radiation enhances the heat transfer process as the total average Nusselt number taking into account surface radiation is higher than that of no radiation case.
3. The increase of Rayleigh number results in a decrease of maximum temperature than attained inside the enclosure.
4. The increase of enclosure aspect ratio has insignificant effect on the radiation Nusselt number whereas the increase of the enclosure aspect ratio increases the convection Nusselt number.
5. The maximum temperature that attained inside the enclosure decreases as the enclosure aspect ratio increases.
6. The emissivity of both cold and adiabatic surfaces has insignificant effect on the radiation Nusselt number.
7. The radiation Nusselt number increases with the increase of the emissivity of the heaters' surfaces whereas the convective Nusselt number decreases with the increase of heater surface emissivity.
8. The increase of radiation parameter leads to an increase of the radiation Nusselt number, a decrease in the convection Nusselt number and an increase of the total Nusselt number.
9. The increase of the radiation parameter leads to a considerable decrease of the maximum temperature that is attained in the enclosure.
10. Useful correlations are obtained for the average Nusselt number, its radiative and convective components as well as the maximum temperature that is attained inside the enclosure in terms of the parameters that govern the phenomenon of surface radiation interaction with the natural convection.

REFERENCES
Fig. 1. (a) Schematic diagram of the physical domain and coordinate system (b) Finite element discretization for the solution domain.

Fig. 2. Comparison between the present model prediction and the experimental data [17].

Fig. 3. Effect of surface radiation on the structures of streamlines (left) and isotherms (right), \( Ra=10^5, \) \( Pr=1.5, \) \( Ar=5, \) \( \epsilon_h=\epsilon_c=\epsilon_a=0.5 \)
INTERACTION OF SURFACE RADIATION WITH NATURAL CONVECTION AIR COOLING OF DISCRETE HEATERS IN A VERTICAL RECTANGULAR ENCLOSURE

![Fig. 4. Effect of surface radiation on the structures of streamlines (left) and isotherms (right), $Ra=5\times10^6$, $N_R=1.5$, $AR=5$, $\varepsilon_h=\varepsilon_c=\varepsilon_a=0.5$.](image1)

![Fig. 5. Effect of surface radiation on the structures of streamlines (left) and isotherms (right), $Ra=5\times10^6$, $N_R=1.5$, $AR=7$, $\varepsilon_h=\varepsilon_c=\varepsilon_a=0.5$.](image2)
Fig. 6. The variation of the average Nusselt number with Rayleigh number for (AR=5, NR=1.5, \( \varepsilon_h=\varepsilon_c=\varepsilon_a=0.5 \)) (a) First heater (b) Second heater (c) Third heater (d) Fourth heater.

Fig. 7. The variation of the maximum dimensionless temperature with Rayleigh number for (AR=5, NR=1.5, \( \varepsilon_h=\varepsilon_c=\varepsilon_a=0.5 \)) (a) First heater (b) Second heater (c) Third heater (d) Fourth heater.
Fig. 8. The variation of the average Nusselt number with aspect ratio for \((Ra=10^6, N_R=1.5, \epsilon_h=\epsilon_c=\epsilon_a=0.5)\)  
(a) First heater    
(b) Second heater    
(c) Third heater    
(d) Fourth heater

Fig. 9. The variation of the maximum dimensionless temperature with aspect ratio for \((Ra=10^6, N_R=1.5, \epsilon_h=\epsilon_c=\epsilon_a=0.5)\).
Fig. 11. The variation of the maximum dimensionless temperature with the radiation parameter for \( (Ra=10^6, \ AR=5, \ \varepsilon_h=\varepsilon_c=\varepsilon_a=0.5) \).
Fig. 12. The variation of the average Nusselt number with the heater emissivity for \((Ra=10^6, \ AR=5, \ NR=1.5, \ \varepsilon_c=\varepsilon_a=0.5)\) (a) First heater (b) Second heater  (c) Third heater  (d) Fourth heater
Fig. 13. The variation of the maximum dimensionless temperature with the heater emissivity for \( (Ra=10^6, AR=5, N_R=1.5, \varepsilon_c=\varepsilon_a=0.5) \).

Fig. 14. The variation of the maximum dimensionless temperature with aspect ratio.

Without radiation \hspace{2cm} \text{With radiation}

Vol. 2, No. 4, Apr. 2007
79