SELECTIVE HARMONIC ELIMINATION PWM VOLTAGE SOURCE INVERTER BASED ON GENETIC ALGORITHM

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Abstract. In this paper, the selective harmonics elimination (SHE) technique is used to eliminate low-order harmonics with relatively low inverter switching frequency. The major challenge which faces this technique is solving large numbers of nonlinear transcendental equations at each modulation index to generate the power inverter switching patterns. Genetic algorithm (GA) technique is used to obtain the optimal switching angles. Simulation and experimental results are obtained to be guarantee that GA is eliminated the desired harmonics.

Keywords: Selective harmonic elimination, pulse width modulation, Genetic algorithm

1. INTRODUCTION

The selective harmonic elimination pulse width modulation (SHE-PWM) technique was first reported in 1964 [1], [2]. The SHE-PWM technique is based on pre-calculating the width of each pulse that eliminates a specific set of low-order harmonics in the inverter output voltage. A set of non-linear equations is formulated and are simultaneously solved to determine the switching instants of the power semiconductor devices. The solution of this set of nonlinear equations is the major challenge which faces this technique. The detail of the mathematical equations of SHE-PWM technique are derived in references [3], [4].

As the SHE-PWM technique is based on simultaneously solving several numbers of nonlinear equations, most of the studies have focused on developing accurate and efficient algorithms to solve these equations. One of the proposed techniques is based on Newton-Raphson method. In this technique, the nonlinear equations are linearized around a predicated solution [3]. The convergences of the solutions rely on a trial and error process. This technique is efficient when it is required to solve relatively small number of nonlinear equations.

Another approach is based on the Walsh function for harmonic elimination of PWM waveforms [5], [6]. This algorithm has the advantages of converting the nonlinear transcendental equations to linear equations. The amplitude of the inverter output harmonics are expressed linearly as functions of switching angles. Generally, Walsh function can’t be applied in some cases of operation [7].

In the mathematical theory of resultants technique, the nonlinear transcendental equations are converted into an equivalent set of polynomial equations [8]. The disadvantage of this algorithm that the order of the polynomial
equations increases as the number of harmonics needs to be eliminated increase. Therefore, it is only suitable when a small number of harmonics needs to be eliminated.

To overcome the disadvantages of the previous algorithms, optimization techniques are used for solving the nonlinear equations. Some of these techniques are such as:

1. Minimization technique, where the equations don’t need to be set to zero, but they are minimized to very small values [2],[9]. However, multiple set of solutions are obtained by this technique. Hence, it is required to select the best solution to obtain the best performance for the inverter.
2. The artificial neural network algorithm [10].
3. Genetic algorithms-based solution [11], [12].
4. Particle swarm optimization [13].

Finally, an approach based on equal area criteria method [14], [15]. It doesn’t involve complicated equations. However, the numbers of equations in this method are four times the nonlinear equations in the other previous methods [15].

From the previous studies, SHE-PWM technique has the following advantages [4]:

- About 50% reduction in the inverter switching frequency is achieved when comparing with SPWM scheme.
- Reduction in the switching frequency which tends to the reduction in the switching losses of the inverter.
- A reduction in the size of the DC input filter is achieved, where the ripples in the DC link current are small
- The possibility of using this technique in over modulation. Hence, this contributes to higher utilization of the power conversion process.
- Elimination of lower-order harmonics reduces the electro-magnetic interference (EMI) with the other circuits at the same network.
- Using pre-calculated switching patterns avoid on line computations.

However, this technique suffers from [2]-[15]:

- Obtaining analytical solutions of nonlinear transcendental equations which contain trigonometric terms that naturally exhibit multiple solutions.
- The optimal switching pattern is determined from a set of solutions.

The details of the mathematical equations of the SHE-PWM will be discussed. Then, the algorithms such as Newton-Raphson and Genetic algorithm will be presented to solve these equations. Finally, a simulation and experimental results will be observed.
2. MATHEMATICAL EQUATIONS OF SHE-PWM INVERTER

SHE-PWM technique is based on chopping the output voltage of the conventional (square-wave) inverter to several pulses of different widths. The widths of these pulses are pre-calculated in such a manner to eliminate any specified number of low-order harmonics. For low and medium power operation, the output voltage waveforms of the inverter may be unipolar or bipolar, while for high power applications the multilevel voltage waveforms are widely used [14], [15].

2.1 Unipolar Pattern

In the unipolar PWM inverters, the output voltage is altered between \(+V_{dc}\) and zero during the positive half cycle and altered between zero and \(-V_{dc}\) for negative half cycle, as shown in Fig. 1. The width of each pulse is dependent on the selected undesired harmonics to be eliminated.

![FIGURE 1. Unipolar PWM output inverter voltage waveform](image)

Fourier series representation of the unipolar output voltage is given by [3],[4], and [8]:

$$V_n = \frac{4V_{dc}}{N\pi} \sum_{k=1}^{N} (-1)^{k+1} \cos(n\alpha_k)$$  (1)
Where:
- $\alpha_k$: $k^{th}$ switching angles, where $\alpha_1 < \alpha_2 < \alpha_3 \ldots < \alpha_k < \pi/2$
- $V_n$: The magnitude of $n$th harmonic component of the inverter output voltage
- $N$: Number of switching angles in the first quarter period.
- $n$: The harmonic order. It is worth mentioning that waveform do not contain even harmonics because it is odd quarter-wave symmetry.

2.2 Bipolar Pattern

In bipolar PWM inverters, the output voltage is either $+V_{dc}$ or $-V_{dc}$ as shown in Fig. 2 and Fig. 3. The Fourier series of the output voltage waveform which shown in Fig. 2 and Fig. 3 has two forms of expressions depending on the number of undesired harmonics to be eliminated [16].

![FIGURE 2. Bipolar SHE-PWM output voltage waveform for eliminating odd number of harmonics](image-url)
For eliminating odd number of harmonics, the PWM pattern of the inverter output voltage is shown in Fig. 2, and Fourier series of the inverter output voltage waveform is given by [16]:

\[
V_n = \frac{4V_{dc}}{n\pi} \left[ 1 + 2 \sum_{k=1}^{N} (-1)^k \cos(n\alpha_k) \right]
\]  

And in the case of eliminating even number of harmonics, the PWM pattern is shown in Fig. 3, and Fourier series is given by [16]:

\[
V_n = \frac{4V_{dc}}{n\pi} \left[ -1 - 2 \sum_{k=1}^{N} (-1)^k \cos(n\alpha_k) \right]
\]

The following section explains some algorithms which are used for solving the previous nonlinear mathematical equations.

3. ALGORITHMS FOR SOLVING NON-LINEAR SHE-EQUATIONS

Many algorithms have been used to obtain the switching patterns of the inverter switches from the previous equations. The most popular algorithms are:

- Newton-Raphson (NR) algorithm
- Genetic Algorithm (GA)
- Practical Swarm Optimization (PSO)
• Artificial Neural Network (ANN)
The first and second types of these techniques are presented in this paper.

3.1 Newton-Raphson Algorithm
Newton-Raphson (NR) algorithm is a linearization technique, where the nonlinear equations are linearized around initial values of the switching angles [3]. The convergence or divergence from the solution depends on the initial guess of the switching angles.

The steps for computing the switching angles are as follow [3]:
1- The nonlinear equations put in the following shape:
\[
\begin{align*}
f_1(\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_N) &= \frac{\pi}{4} M_i \\
f_2(\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_N) &= 0 \\
&\quad \text{......} \\
f_N(\alpha_1,\alpha_2,\alpha_3,\ldots,\alpha_N) &= 0
\end{align*}
\]
(4)

Where, \( M_i \) is the modulation index.
2- Guess a set of initial values for \( \alpha \)
\[
\alpha^0 = [\alpha_1^0,\alpha_2^0,\alpha_3^0,\ldots,\alpha_N^0]
\]
(5)
3- Determine the value of (4) at \( \alpha^0 \)
\[
f(\alpha^0) = f^0
\]
(6)
4- Linearize (4) around \( \alpha^0 \) by the equation:
\[
f^0 + \left[ \frac{\partial f}{\partial \alpha} \right]^0 \Delta \alpha = 0
\]
(7)

Where \( \left[ \frac{\partial f}{\partial \alpha} \right]^0 \) is the Jacobian matrix and it is given by:
\[
\left[ \frac{\partial f}{\partial \alpha} \right] = 
\begin{bmatrix}
\frac{\partial f_1}{\partial \alpha_1^0} & \frac{\partial f_1}{\partial \alpha_2^0} & \cdots & \frac{\partial f_1}{\partial \alpha_N^0} \\
\frac{\partial f_2}{\partial \alpha_1^0} & \frac{\partial f_2}{\partial \alpha_2^0} & \cdots & \frac{\partial f_2}{\partial \alpha_N^0} \\
&\cdots&\cdots&\cdots \\
\frac{\partial f_N}{\partial \alpha_1^0} & \frac{\partial f_N}{\partial \alpha_2^0} & \cdots & \frac{\partial f_N}{\partial \alpha_N^0}
\end{bmatrix}
\]
(8)
5- Solving (7) to obtain $\Delta \alpha$ then,

$$\alpha^{\text{new}} = \alpha^{\text{old}} + \Delta \alpha$$

(9)

6- Repeat the steps from 1 to 5 until (9) reaches to the desired value of accuracy.

The flow chart of this algorithm is shown in Fig. 4.

![Newton-Raphson flow chart for solving SHE-PWM equations](image)

**FIGURE 4.** Newton-Raphson flow chart for solving SHE-PWM equations

Where $\text{iter}_{\text{max}}$ is the maximum number of iterations and $\varepsilon$ is the desired degree of accuracy.

Although this algorithm is simple, it has the following disadvantages:

(i) Rapid convergence of the solution strongly depends on the initial guess of the switching angles.

(ii) The step of determining the Jacobin matrix is repeated each iteration. Therefore, this method is limited in case of eliminating relatively large number of harmonics.

3.2 Genetic Algorithm

Genetic algorithm (GA) is based on the genetics and development mechanisms observed in nature system and population of living beings [11]. GA is non-traditional programming technique which used to give a set of solutions for an objective function. Therefore, GA method is employed to obtain the most optimal
switching patterns of the inverter voltage by solving SHE-PWM mathematical equations without initial guess of the switching angles values. However, these values are generated randomly by the program [12].

The GA is used to solve the SHE-PWM equations. Consider the equation (2) as an example. The procedure will be as follow:

1. Expansion of (2) gives the fundamental and harmonic components of the output voltage of the single-phase inverter as:

\[
\begin{align*}
    f_1(\alpha) &= \frac{4}{\pi} \left[ 1 + 2 \sum_{k=1}^{N} (-1)^k \cos(\alpha_k) \right] - M_i = \varepsilon_1 \\
    f_2(\alpha) &= \frac{4}{3\pi} \left[ 1 + 2 \sum_{k=1}^{N} (-1)^k \cos(3\alpha_k) \right] = \varepsilon_2 \\
    f_3(\alpha) &= \frac{4}{5\pi} \left[ 1 + 2 \sum_{k=1}^{N} (-1)^k \cos(5\alpha_k) \right] = \varepsilon_3 \\
    \vdots & \\
    f_N(\alpha) &= \frac{4}{N\pi} \left[ 1 + 2 \sum_{k=1}^{N} (-1)^k \cos(n\alpha_k) \right] = \varepsilon_N
\end{align*}
\]  

Where \( \varepsilon_1 \) to \( \varepsilon_N \) are the normalized values of amplitude of the specific harmonics to be eliminated. The values of \( \varepsilon_1 \) to \( \varepsilon_N \) are set to very small values. The GA is used to minimize an objective function. The objective function of the SHE-PWM problem is:

\[
f_1^2(\alpha) + f_2^2(\alpha) + \ldots + f_N^2(\alpha) \tag{11}
\]

The values of the switching angles are determined depending on minimization of (11) and the inequality constraint matrix (12) is satisfied.

\[
\begin{bmatrix}
    1 & -1 & 0 & 0 & 0 & \ldots & 0 \\
    0 & 1 & -1 & 0 & 0 & \ldots & 0 \\
    0 & 0 & 1 & -1 & 0 & \ldots & 0 \\
    \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    \alpha_1 \\
    \alpha_2 \\
    \alpha_3 \\
    \vdots \\
    \alpha_N
\end{bmatrix}
\leq
\begin{bmatrix}
    0 \\
    0 \\
    0 \\
    \vdots \\
    \frac{\pi}{2}
\end{bmatrix}
\tag{12}
\]
The GA implementation can be summarized in the following steps [11], [12]:

1. At modulation index equal 0.1, the initial populations of switching angles are randomly generated to define different numbers of the initial switching angles where they must satisfy the constraints in (12), and they are uniformly distributed between the limits of maximum ($\pi/2$) and minimum (0) switching angles. This step is called initialization.

2. Calculate the objective function with the previous switching angles populations. If any population has minimized the objective function, the program will be terminated and the switching angles are determined. If the objective function is still large, create new generation of populations switching angles after applying some operators. This selection scheme is known as Roulette wheel selection.

3. The next step of GA is named as “crossover”. Crossover combines two individual populations to form a new individual population for the next generation. There are many functions used to perform the crossover such as, scattered, single point, and two points. A single point crossover method is used to combine the older populations to produce the new population.

4. For the population that gives excluded value, it can make small random changes in that individual’s population to enable the genetic algorithm to search a broader space. This step is called “mutation” operation.

5. The above procedure (from 1 to 4) is repeated until the maximum iteration count ($n_c$) is reached or the objective function is minimized. After reaching a solution, repeat the same procedure for the next value of modulation index. The main stages of the proposed GA technique including initialization, selection, crossover and mutation are shown in the following flow chart of Fig. 5.
To obtain the optimum solution for the given SHE-PWM equations, the following GA parameters were used [11], [12]:
- Population size: 50
- Number of generations ($n_{gen}$): 100
- Crossover rate: 0.8
- Mutation rate: 0.01

4. COMPARATIVE ANALYSIS

To compare between the two selected algorithms that have been described in the previous section, the algorithms are used for obtaining the solution of SHE-PWM mathematical models for eliminating four harmonics in case of bipolar waveform and at 0.8 modulation index.

The relationship between the values of the maximum error at each iteration is shown in Fig. 6. The comparison between both methods is shown in TABLE 1., where the solution of the equations is obtained after 616 iterations when NR
algorithm is used and 57 iterations when GA is used. The error values are 3.29e-5 and 1.49e-7 in case of NR and GA algorithms respectively.

![Graph showing error values](image)

**FIGURE 6.** The value of maximum error at each iteration for GA and NR algorithms

**TABLE 1.** Comparison between GA and NR methods

<table>
<thead>
<tr>
<th>Technique comparison</th>
<th>GA</th>
<th>NR $(a_{initial}=20,40,60,70,75)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum error</td>
<td>1.49e-7</td>
<td>3.29e-5</td>
</tr>
<tr>
<td>Number of iterations</td>
<td>57</td>
<td>616</td>
</tr>
<tr>
<td>Switching angles $a_1$ to $a_5$</td>
<td>23.111671, 33.746607, 47.714661, 68.4750941, 76.455619</td>
<td>23.10192, 33.738097, 47.711828, 68.483316, 76.4669111</td>
</tr>
</tbody>
</table>

**5. SIMULATION AND EXPERIMENTAL RESULTS**

In this paper, the simulation and experimental results of single-phase inverter using SHE-PWM technique based on GA to obtain the desired switching angles are presented. The simulation results are used to verify that the GA is eliminated.
the desired harmonics and then it validated experimentally. The results are obtained for unipolar and bipolar pattern to eliminate 13 low-order odd harmonics, from the harmonic order 3 to the order 27. Number of the switching angles in the first quarter cycle are 14. Hence, number of nonlinear transcendental equations are 14 [16].

5.1 Results of Bipolar Pattern

Relationships between the switching angles and the modulation index is shown in Fig. 7. Figure 7 shows that GA could obtain the optimal switching angles at any modulation index. However, at higher modulation index, two consecutive switching angles are very close to each other so it will need faster microcontroller.

![Figure 7. Bipolar switching angles for eliminating 13 low-order odd harmonics](image)

The simulation and experimental waveform for the inverter output voltage is shown in Fig. 8 at modulation index equal 0.8. To validate that the desired harmonics are eliminated, the fast Fourier transform (FFT) is applied for the inverter output voltage shown in Fig. 9 for simulation and experimental waveforms. It shows that the order of the first harmonic appear in the inverter

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output voltage is the 29\textsuperscript{th} in both the simulation and experimental results. In addition, the magnitude of the harmonics in both cases are nearly equal. However, in the experimental result there is relatively small magnitude in the desired eliminated harmonics. The slight difference between the two spectrums arises from:

1. The non-ideality of the system
2. The dead time required for the IGBT switches to prevent the switch shoot through
3. Turning- on and turning-off time required for the IGBT.

(a) Simulation result
FIGURE 8. Inverter output voltage for bipolar pattern in case $M_i=0.8$
5.2 Result of Unipolar Pattern

Relationships between the switching angles and the modulation index is shown in Fig. 10. Figure 10 shows that GA obtains the optimal switching angles at any modulation index for unipolar pattern. Unlike the bipolar pattern, it doesn’t need faster microcontroller for any modulation index.
FIGURE 10. Unipolar switching angles for eliminating 13 low-order harmonics

The simulation and experimental waveform for the inverter output voltage at modulation index equal 0.8 is shown in Fig. 11. To validate that the desired harmonics are eliminated, the fast Fourier transform is applied for the inverter output voltage shown in Fig. 12 for simulation and experimental waveforms. It shows that the order of the first harmonic appear in the inverter output voltage is the 29th in both the simulation and experimental results. In addition, the magnitude of the harmonics in both cases are nearly equal and less than the harmonics in bipolar pattern. However, in the experimental result there is relatively small magnitude in the desired eliminated harmonics as discussed before in case bipolar pattern.
FIGURE 11. Inverter output voltage for unipolar pattern at $M_i=0.8$
(a) Simulation harmonic spectrum

(b) Experimental harmonic spectrum

FIGURE 12. Inverter output voltage for unipolar pattern at Mi=0.8
CONCLUSION

In this paper, selective harmonic elimination based on Genetic algorithm has been investigated. The algorithm has been used to eliminate 13 low-order harmonics for single-phase inverter with unipolar and bipolar pattern. The simulation and experimental results prove that is GA considered one of the best technique to obtain the optimal switching angles for SHE technique.

REFERENCES