Sheet (3)-solution

1. The maximum radiation intensity of a 90% efficiency antenna is 200 mW/ unit solid angle. Find the directivity and gain (dimensionless and in dB) when the
(a) Input power is 125.66 mW
(b) Radiated power is 125.66 mW

\[
D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi (200 \times 10^{-3})}{0.9 \times (125.66 \times 10^{-3})} = 22.22 = 13.47 \text{ dB}
\]
\[
G_0 = \varepsilon \cdot D_0 = 0.9 \times (22.22) = 20 = 13.0 \text{ dB}
\]

\[
D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi (200 \times 10^{-3})}{(125.66 \times 10^{-3})} = 20 = 13.0 \text{ dB}
\]
\[
G_0 = \varepsilon \cdot D_0 = 0.9 \times (20) = 18 = 12.55 \text{ dB}
\]

2. A lossless resonant half-wavelength dipole antenna, with input impedance of 73 ohms, is connected to a transmission line whose characteristic impedance is 50 ohms. Assuming that the pattern of the antenna is given approximately by \( U = B_0 \sin^3 \theta \). Find the maximum gain and maximum absolute gain of this antenna.

\[
U_{\text{max}} = U_{\text{max}} = B_0
\]
\[
P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi U(\theta, \phi) \sin \theta d\theta d\phi = 2\pi B_0 \int_0^\pi \sin^4 \theta d\theta = B_0 \left( \frac{3\pi^2}{4} \right)
\]
\[
D_0 = 4\pi \frac{U_{\text{max}}}{P_{\text{rad}}} = \frac{16}{3\pi} = 1.697
\]

Since the antenna was stated to be lossless, then the radiation efficiency \( e_{cd} = 1 \).
\[
G_0 = e_{cd} D_0 = 1 \times (1.697) = 1.697
\]
\[
e_r = (1 - |\Gamma|^2) = \left( 1 - \frac{|73 - 50|^2}{|73 + 50|^2} \right) = 0.965
\]
\[
G_{\text{abs}} = e_0 D_0 = 0.965 \times (1.697) = 1.6376
\]

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3. A uniform plane wave, of is traveling in the positive z-direction. Find the polarization (linear, circular, or elliptical), sense of rotation (CW or CCW), when
(a) \( Ex = Ey, \Delta \phi = \phi_y - \phi_x = 0 \)
(b) \( Ex \neq Ey, \Delta \phi = \phi_y - \phi_x = 0 \)
(c) \( Ex = Ey, \Delta \phi = \phi_y - \phi_x = \pi/2 \)
(d) \( Ex = Ey, \Delta \phi = \phi_y - \phi_x = -\pi/2 \)
(e) \( Ex = Ey, \Delta \phi = \phi_y - \phi_x = \pi/4 \)
(f) \( Ex = Ey, \Delta \phi = \phi_y - \phi_x = -\pi/4 \)
(g) \( Ex = 0.5Ey, \Delta \phi = \phi_y - \phi_x = \pi/2 \)
(h) \( Ex = 0.5Ey, \Delta \phi = \phi_y - \phi_x = -\pi/2 \)

4. A wave traveling normally outward from the page (toward the reader) is the resultant of two elliptically polarized waves, one with components of \( E \) given by:
\[
\varepsilon_y' = 3 \cos \omega t
\]
\[
\varepsilon_x' = 7 \cos \left( \omega t + \frac{\pi}{2} \right)
\]
And the other with components given by:

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5. Design an antenna with omnidirectional amplitude pattern with a half-power beam width of 90°. Express its radiation intensity by $U = \sin^n \theta$. Determine the value of $n$ and attempt to identify elements that exhibit such a pattern. Determine the directivity of the antenna.

**Solution:** Since the half-power beamwidth is 90°, the angle at which the half-power point occurs is $\theta = 45°$. Thus

$$U(\theta = 45°) = 0.5 = \sin^n(45°) = (0.707)^n$$

or

$$n = 2$$

$$U_{\text{max}} = 1$$

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^{\pi} \sin^2 \theta \sin \theta \, d\theta \, d\phi = \frac{8\pi}{3}$$

$$D_0 = \frac{4\pi}{8\pi/3} = \frac{3}{2} = 1.761 \, \text{dB}$$

6. The normalized far-zone field pattern of an antenna is given by

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Find the directivity using
(a) The exact expression
(b) Kraus’ approximate formula

\[ E = \begin{cases} \left(\sin \theta \cos^2 \phi\right)^{1/2} & 0 \leq \theta \leq \pi \text{ and } 0 \leq \phi \leq \pi/2, \ 3\pi/2 \leq \phi \leq 2\pi \\ 0 & \text{elsewhere} \end{cases} \]

\[ U = \frac{1}{2\pi} |E|^2 = \frac{1}{2\pi} \sin \theta \cos^2 \phi \Rightarrow U_{\text{max}} = \frac{1}{2\pi} \]

\[ (a). \quad P_{\text{rad}} = 2 \int_0^{\pi/2} \frac{1}{\pi} \sin \theta \cos^2 \phi \, d\theta \, d\phi = \frac{1}{\pi} \left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\pi} \]

\[ D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi \left(\frac{1}{2\pi}\right)}{\frac{\pi^2}{8\pi}} = \frac{16}{\pi} = 5.09 \text{ dB} \]

\[ (b). \quad U_{\text{max}} = \frac{1}{2\pi} \sin \theta \cos^2 \phi \Rightarrow \theta = \pi/2, \phi = 0 \]

In the elevation plane through the maximum \( \theta = 0 \) and \( U = \frac{1}{2\pi} \sin \theta \). The 3-dB point occurs when

\[ U = 0.5 U_{\text{max}} = 0.5 \left(\frac{1}{2\pi}\right) = \frac{1}{2\pi} \sin \theta \Rightarrow \theta = \sin^{-1}(0.5) = 30^\circ \]

Therefore \( \theta_{\text{d1}} = 2(90^\circ - 30^\circ) = 120^\circ \)

In the azimuth plane through the maximum \( \theta = \pi/2 \) and \( U = \frac{1}{2\pi} \cos^2 \phi \). The 3-dB point occurs when

\[ U = 0.5 U_{\text{max}} = 0.5 \left(\frac{1}{2\pi}\right) = \frac{1}{2\pi} \cos^2 \phi \Rightarrow \phi = \cos^{-1}(0.707) = 45^\circ \]

Therefore using Kraus’ formula \( D_0 \approx 4.1253 = 3.82 = 5.82 \text{ dB} \)

\[ \theta_{\text{d1}} = 2(90^\circ - 45^\circ) = 45^\circ \]

\[ \theta_{\text{d1}} = 2(90^\circ - 45^\circ) = 45^\circ \]

REPORT

1. The normalized radiation intensity of an antenna is represented by

\[ U(\theta) = \cos^2(\theta) \cos^2(3\theta), \quad (0 \leq \theta \leq 90^\circ, \ 0^\circ \leq \phi \leq 360^\circ) \]

Find the exact and approximate directivity.

2. The radiation intensity is represented by

\[ U = \begin{cases} U_0 \sin(\pi \sin \theta), & 0 \leq \theta \leq \pi/2 \text{ and } 0 \leq \phi \leq 2\pi \\ 0 & \text{elsewhere} \end{cases} \]

Find \( \theta_{\text{HP}} \) and draw the radiation pattern.

\[ \text{Good Luck} \]

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