OPTIMIZING ANALYSIS FOR RECTANGULAR STEP ISOLATED FOUNDATION ON SAND SOIL USING FINITE ELEMENT METHOD

by

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Abstract

The behaviour of a rectangular step isolated reinforced concrete foundation is studied. The effects of changing foundation rectangularity, depth, and pedestal dimensions under column on settlement, contact soil pressure, and concrete stresses are considered. The Finite Element Method using a three dimensional eight nodes solid element is applied with the concept of linear elastic Winkler spring to represent the vertical stiffness of soil foundation. The results showed that using this type of foundation a remarkable reduction in concrete volume is achieved. In the most cases of analysis an acceptable increase in deflections, contact soil pressure, and concrete stresses occurred. Graphs for optimizing the concrete volume are plotted, leading to a considerable reduction in the foundation cost.

Introduction

Conventional isolated reinforced concrete foundations of constant depth were studied earlier by many researchers as explained in Reference (1). Many studies were carried out on the elastic raft foundations (2 and 3), where the Finite Element Method (1,4 and 5) was used to analyze raft foundations. The Finite Element Method was also applied to analyze square step isolated foundation (6), where a considerable results were obtained.

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The aim of this paper is to study the relationship between increasing deflection, contact pressure, and concrete stress and reduction in concrete volume as a result of changing foundation rectangularity, depth, and pedestal dimensions under column as shown in Figure (1). In this study, the Finite Element Method is used with the concept of linear elastic Winkler spring to represent the vertical stiffness of soil foundation.

Modulus of Subgrade Reaction

The ratio between unit soil pressure and the corresponding settlement is the modulus of subgrade reaction. The value can be determined by performing a plate-load test.

Terzaghi (1955)(1) proposed that modulus of subgrade reaction (k_S) for full size footing on sand could be obtained from plate-load test using the following equation:

$$k_s = k_1\left(\frac{B+1}{2B}\right)^2 \quad \text{..........(1)}$$

in which, (k_1) is the value from a *1x1 ft* square load test.

Vesic (1961) proposed that the modulus of subgrade reaction could be computed using the stress-strain modulus from triaxial test as,

$$k'_s = 0.65\sqrt[12]{\frac{E_s B^4}{E_f I_f}}\,\frac{E_s}{1-\mu^2} \quad \text{..........(2)}$$

where E_s, E_f = modulus of soil and footing , respectively, in consistent units

B, I_f = footing width and its moment of inertia based on cross section in consistent units., then (k_s) can be obtained from (k'_s) as,

$$k_s = \frac{k'_s}{B} \quad \text{..........(3)}$$

It was found (1) that bending moment and the computed soil pressure are not very sensitive to what is used for (k_s). This because the structural member stiffness is usually 10 or more times as greater as the soil stiffness as measured by (k_s). Bowels(1) suggested the following relation for approximating (k_s) for sand soil using the allowable bearing capacity (q_a),

$$k_s = 40(SF)q_a \quad \text{kN/m}^3 \quad \ldots\ldots(4)$$

where, *SF* is the safety factor.

Analysis and Finite Element Mesh

The finite element mesh is as shown in Figure (2a), where a three dimensional eight-nodes solid element is used. The mathematical approach of this analysis is as explained in Reference (4). A quarter of the foundation is used for the analysis as a result of double symmetry.

The foundation rectangularity is considered by use a factor (ρ), which is called a rectangularity factor where,

$$\rho = l / L = b / B \quad \ldots\ldots(5)$$

The foundation is divided into four horizontal layers each of one fourth of the total depth (*D*), where the depth factor (δ) is as follows,

$$\delta = d / D \quad \ldots\ldots(6)$$

Also, the dimension of the pedestal under the column is governed by the breadth factor (β) as shown in Figure (1) where,

$$\beta = b / B \quad \ldots\ldots(7)$$

The analysis is carried out for depth factor (δ) equals to ***1.0, 0.75, 0.5,*** and ***0.25***, while the values of breadth factor (β) are chosen as ***1.0, 0.75, 0.5, 0.4,*** and ***0.3***. The ratio β=***0.3*** is the minimum value required for punching requirement. The conventional case occurs when (δ) or (β) or both equal to unity.

One fourth of the column load is idealized to be concentrated and proportionally weighted at nine nodes as shown in Figure (2b). The linear elastic Winkler spring to represent the vertical stiffness of soil foundation is calculated from equation (4).

To carry out the analysis producing non-dimensional relationship between parameters a *square reference foundation* is chosen with breadth B=3.3 m, depth $D = 0.6$m, and column cross-section of 0.45x0.45 m The column load is P=2400 kN.

Other two foundations are chosen according to a factor called reference area factor (α) where,

$$\alpha = A / A_o \qquad \text{.........(8)}$$

in which, (A) is the area of foundation under consideration and (A_O) is the area of the reference foundation mentioned above. The first foundation is chosen with a reference area factor (α=*0.5*). Therefore, its breadth $B = 2.35$m, depth $D = 0.45$m, and column cross-section is 0.3x0.3m. The column load is P=1200 kN. The depth is calculated to produce a close shear stress to that calculated from punching condition in reference foundation. The second foundation is chosen with a reference area factor (α=2). Its dimensions are $B = 4.70$m, $D = 0.85$m, and column cross-section equals 0.65x0.65m. The column load is P=4800 kN.

The analysis was carried out using a rectangularity factor (ρ) equals 0.75 and 0.50 where the square foundation of ρ = 1.0 was studied in reference (6). The pervious three foundations have the following dimensions,

Table (1) Foundation Dimensions and Loads for Different Values of (ρ) and (α)

Factor (ρ)	**Factor (α)**	**L** *m*	**B** *m*	**wx** *m*	**wy** *m*	**D** *m*	**Column Load P**
ρ = 0.75	α = 2.0	*5.40*	*4.05*	*0.73*	*0.55*	*0.85*	*4800kN*
	α = 1.0	*3.80*	*2.85*	*0.52*	*0.39*	*0.60*	*2400kN*
	α = 0.5	*2.70*	*2.00*	*0.35*	*0.26*	*0.45*	*1200kN*
ρ = 0.50	α = 2.0	*6.6*	*3.3*	*0.90*	*0.45*	*0.85*	*4800kN*
	α = 1.0	*4.65*	*2.35*	*0.64*	*0.32*	*0.60*	*2400kN*
	α = 0.5	*3.30*	*1.65*	*0.450*	*0.225*	*0.45*	*1200kN*

The concrete properties are modulus of elasticity E =21700000 kN/m^2 and Poisson ratio μ=0.3. The soil properties are q_a=225 kN/m^2, and k_s=18000 kN/m^3.

Results and Discussion

The changing in relative concrete tensile stresses in X and Y-directions for nodes coincide with Y and X axes respectively are shown Figures (3 to 10). The relative values are calculated with respect to the stress at foundation center at node (1), which is calculated from the conventional case (i.e. β= 1.0 and δ= 1.0). The stress at Node (1) is used as reference value because it is the maximum stress occurs at the foundation center which is not exceeding the working stress. The percentage decrease of concrete volume corresponding to different values of factors (δ) and (β) is as shown in Table (2) and Figure (11).

Figures (3 to 6) indicate the effect of changing depth factor on concrete stresses for rectangularity factor ρ = 0.75 and 0.5 with a reference area factor α = 1.0. Firstly the changing in stresses in X-direction, i. e. normal on shorter side, is shown in Figures (3 and 5) for rectangularity factor ρ = 0.75 and 0.50 respectively. In most cases the increasing of stresses reached up to 10% for the case of ρ = 0.75 as shown in Figures (3a to c), which are corresponding to decreasing in concrete volume up to 63% for the case of δ = 0.25 and β = 0.4 as shown in Table (2). The stress increasing could be reached up 20% as occurred in case of δ= 0.5 and β= 0.3 as shown in Figure (3b) with a decreasing in concrete volume by 45.5% as shown in Table (2). The reinforcement in this direction will be increased by the same ratio, because the stresses are actually a tensile stresses. Obviously, the increase of reinforcement is not needed across the hole side of foundation but it is only needed at the area where relative stresses increase unity. For instance as shown in Figure (5b), the distances required for β =0.75 and 0.4 are 20% and 45% respectively of shorter foundation side below the column. Therefore, in this case an economic design should be carried out to compare the increasing in reinforcement and decreasing in concrete volume costs. The influence of changing rectangularity is shown in Figures (3 and 5) for ρ = 0.75 and 0.5 respectively, where the increasing of stresses is nearly double when changing rectangularity from 0.75 to 0.50. On other hand the increasing in stresses for ρ= 1.0 (square foundation) is up to about 5% as studied in Reference (6).

The stresses in Y-direction, i.e. normal on longer side, are plotted as shown in Figure (4 and 6) for rectangularity factor ρ=0.75 and 0.5 respectively for area factor $\alpha = 1.0$. In most cases the stresses are below the stress at Node (1) except the case when $\delta =$ 0.50 and $\beta = 0.3$ or $\delta = 0.25$ and $\beta = 0.3$ or 0.4 as shown in Figures (4b and c, and 6b and c).

The changing of stresses as a results of changing rectangularity is shown in Figures (7 and 8, and 9 and 10) for area factor $\alpha = 2.0$ and 0.50 respectively for a depth factor δ = 0.25 where stresses become more sensitive at this depth factor. It is observed that in X-direction the stresses become closer for different values of (β) while the rectangularity decreases as shown in Figures (7 and 9). But on Y-direction, the stresses increase while rectangularity decreases as shown in Figures (8 and 10). Also all cases produce stresses below working stress except for few cases as shown in Figures (8 b and c, and 10 b and c). Generally, the reinforcement distribution of the conventional foundation is usually uniform. As shown in Figures (4 and 6), decreasing depth factor increases the stress but in the most cases these stresses are still smaller the working stress. Therefore it could be concluded that in this direction the changing of rectangularity has a limited effect on the reinforcement normal on the longer foundation side. The effect of the area factor on stresses could be observed as shown in Figures (7 to 10). The stresses become closer as the area factor increases, because the smaller the area factor the smaller the depth and in turn the smaller bending rigidity producing higher bending stresses.

The change of settlement at foundation centered at Node (1) as a results of changing foundation dimensions is plotted as shown in Figures (12 and 13) for rectangularity factors 0.75 and 0.50 respectively and different values of area, depth, and breadth factors. Results for rectangularity factor ρ = 1.0 was analyzed as explained in Reference (6).The percentage of increasing settlement is calculated with respect to settlement of corresponding conventional foundation at Node (1). As far as the subgrade modulus of reaction is a linear relationship the percentage increase of contact soil pressure is similar to that settlement increase as shown in Figures (12 and 13). It is observed that settlement increases as rectangularity factor (ρ) decreases. Also it increases as depth factor (δ) and breadth factor (β) decrease. But on other hand, the settlement decreases as area factor (α) decreases. This is because for the same soil pressure the smaller the foundation area the smaller the soil volume affected by stresses leading to smaller settlement.

The results emphasize that from the practical view a considerable volume could be saved as shown in Table (2) and Figure (11), while the settlement or contact soil pressure or reinforcement is increased by an acceptable ratio, especially it is well known that the settlement is usually within a few centimeters. However the concrete stresses increase as a result of decreasing concrete volume, but these stresses in most of cases are still very close to the working stresses at node (1) or less which means no further reinforcements are needed. In some cases where an additional reinforcement is needed an economic comparison for design should be done. Also it is noticed that, when the breadth factor β=*0.75* the solution is very close to the conventional case. Therefore it is not suggested values for (β) greater than 0.75 .

To emphasize the importance of these Graphs, suppose it is required to economize a foundation has column load P = 2400 kN, with column dimensions of 0.55x0.375m The maximum permissible increase in settlement or soil pressure is 5%, then estimate the percentage increase in foundation reinforcement. Referring to the reference foundation then the reference area factor for current foundation is α = 1.0. The rectangularity of foundation is used as that of column, then ρ = 0.375/0.55 = 0.68. The following available cases are,

Case (1): From Figure (12) where (ρ = 0.75), then

a): For (δ=0.5) at (α=1.0) with 5% increase in settlement, Then (β=0.4)

From Figure (11) Corresponding reduction in concrete volume is about 0.42%

From Figure (5b) corresponding increase in concrete stress is about 14% for a distance 30% of the shorter side of foundation below the column

b): For (δ=0.25) at (α=1.0) with 5% increase in settlement, Then (β=0.63)

From Figure (11) corresponding reduction in concrete volume is about 0.45%

From Figure (5c) corresponding increase in concrete stress is about 6% for a distance 20% of the shorter side of foundation below the column

Case (2): From Figure (13) where (ρ = 0.50), then

a): For (δ=0.5) at (α=1.0) with 5% increase in settlement, Then (β=0.5)

From Figure (11) corresponding reduction in concrete volume is about 0.37%

From Figure (7b) corresponding increase in concrete stress is about 16% for a distance 40% of the shorter side of foundation below the column

b): For (δ=0.25) at (α=1.0) with 5% increase in settlement, Then (β=0.65)
From Figure (11) corresponding reduction in concrete volume is about 0.41%
From Figure (7c) corresponding increase in concrete stress is about 8% for a distance 35% of the shorter side of foundation below the column

Therefore the best dimensioning cases are (1-b) and (2-b). By use interpolation process using rectangularity factors, the best dimensioning for the current foundation is δ = 0.25 and b = 0.64 with a reduction 42% in concrete volume. The increase of concrete stress and in turn reinforcement is about 7% for a distance equals 24.2% of shorter foundation side below the column. In other words, the average increase in reinforcement for the hole side is 1.69% which is in fact a small amount. The same foundation is solved by the computer program using (δ = 0.25 and β = 0.64) to check the validity of the these Graphs. It is found that the relative percentage increase of concrete stress is (7.3%) which is close for that estimated from Graphs.

Conclusion

Results showed that in most cases a small increase in settlement , soil pressure, and concrete stress occurred with a remarkable decrease in foundation,s volume. The decreasing could be reached up to 60%. This leads to a considerable reduction in the foundation cost. Reducing a foundation depth up to 75% whatever the pedestal dimensions or reducing pedestal dimensions up to 75% whatever the reduction in foundation depth produces a very close solution to the conventional foundation solution. On the other hand, it is not recommended to use a depth simultaneously with pedestal dimensions each smaller than 50% of the conventional dimensions. This leads to a convenient solution from the stress analysis point of view. It is showed that the narrower the foundation the higher the stresses normal to the short side of foundation for the same foundation area. Graphs for optimizing the concrete volume are plotted.

The form-work of this sort of foundation may be more expensive than that made by conventional way. But the total saving in cost when considering the reduction of material emphasizes the importance of the proposed approach of step foundation design from the economical point of view.

References:

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(2): Payne,D.C. "Three-Dimensional Analysis of Stiffened Raft Footing on Swelling Clay Soil" Transaction of the Institution of Engineering, Astralia: Civil Engineering v CE33 n 3 Jul. 1991 P 159-168.

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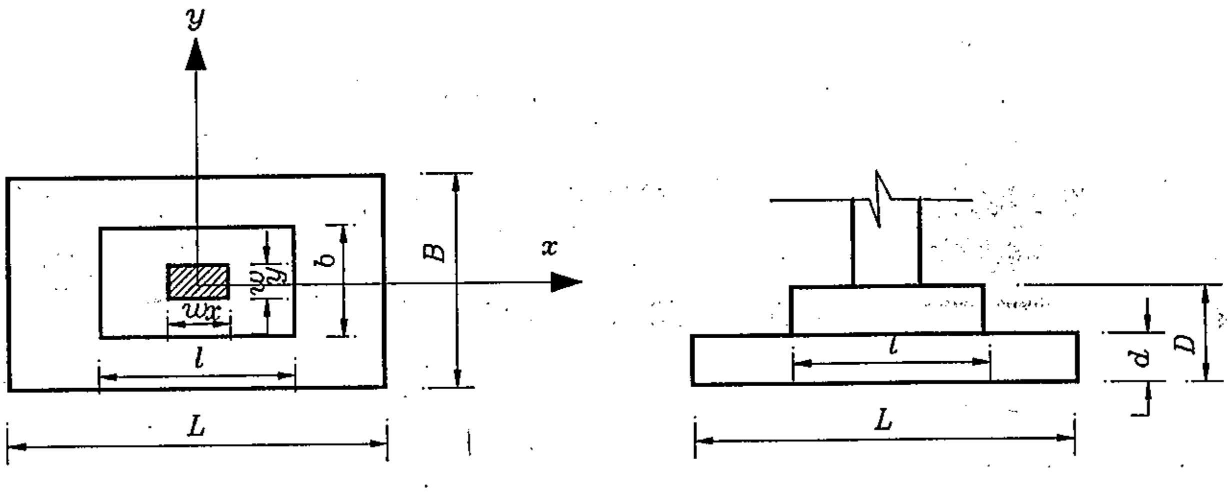
(5): Habibullah,A. and Wilsonm E.L. "Sap90-Structural Analysis Verification Manual, Computers and Structures". Inc, Berkley, California (1992).

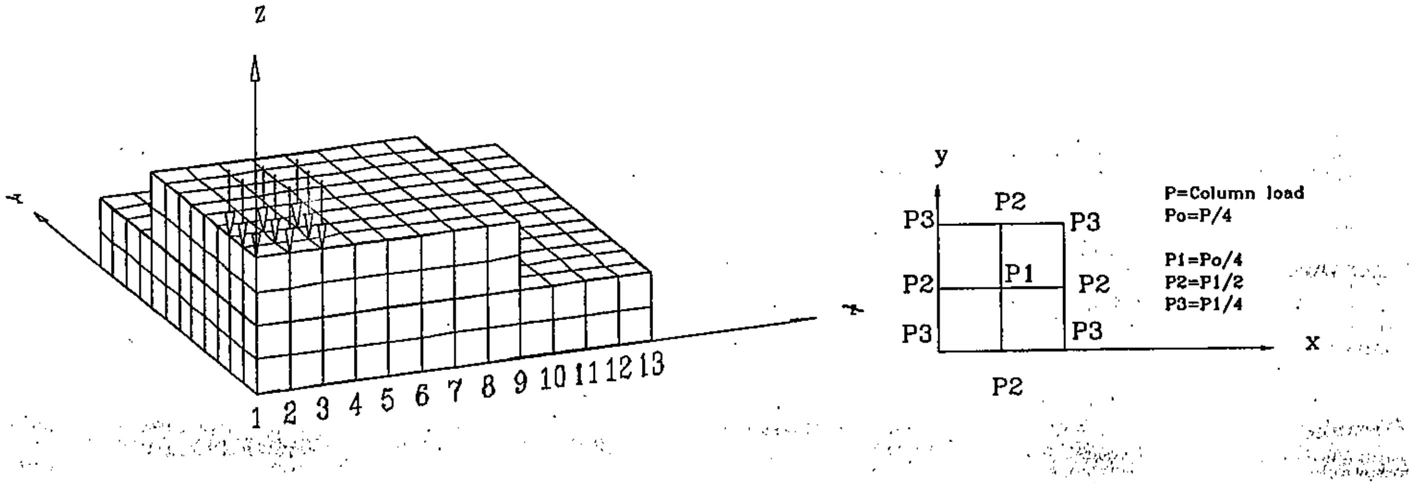
(6): EL-DIB, Mohamed E. Abou-Hashem "Finite Element Anslysis of Elastic Step Foundation on Sand Soil",University of Helwan, Engineering Reseash Bulletin, Vol. 1, Jan. 1994, pp 215-225.



a)- Plan

b)- elevation

Figure (1) Rectangular Foundation with Pedestal



a)- Finite Element Mesh

b)- Idealization Load

Figure (2) Finite Element Load and Column Load Idealization

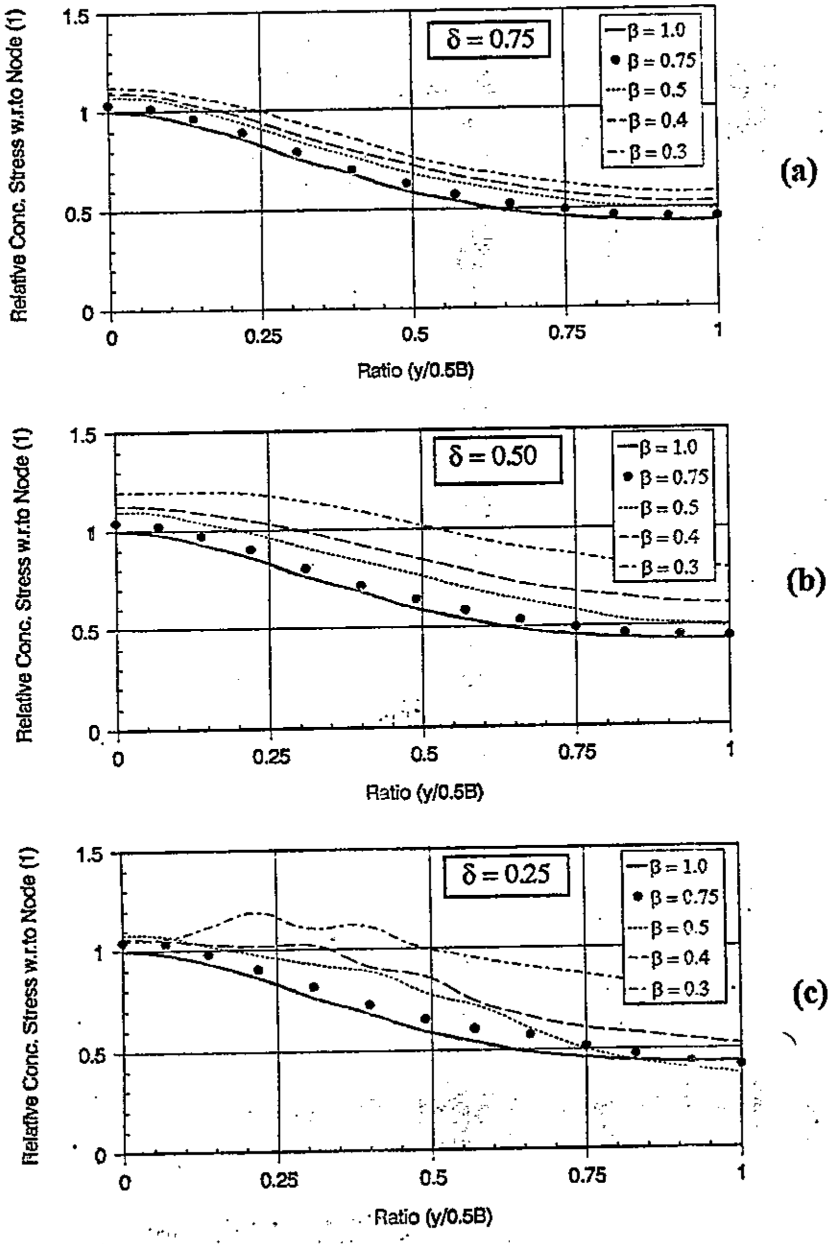


Figure (3) relative Stress in X-Direction For Parameters
$\rho = 0.75$ and $\alpha = 1.0$

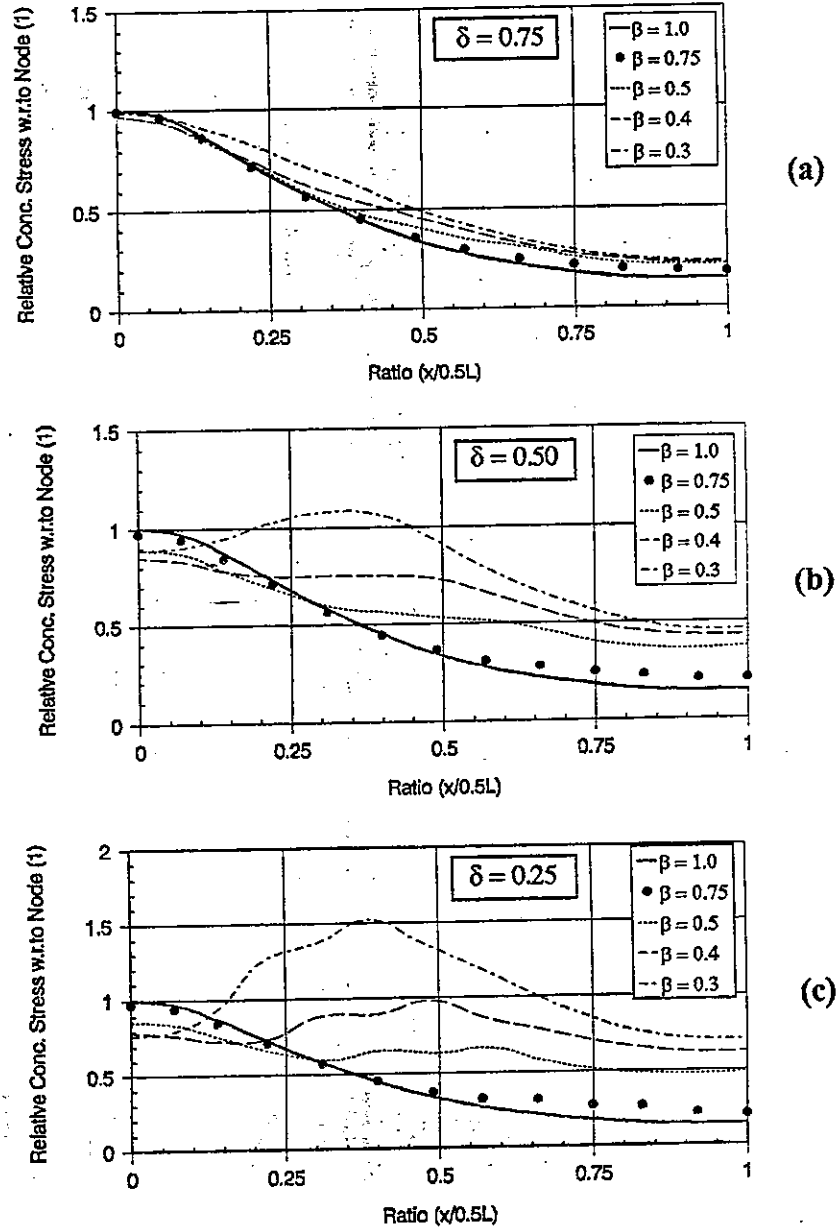


Figure (4) relative Stress in Y-Direction For Parameters
$\rho = 0.75$ and $\alpha = 1.0$

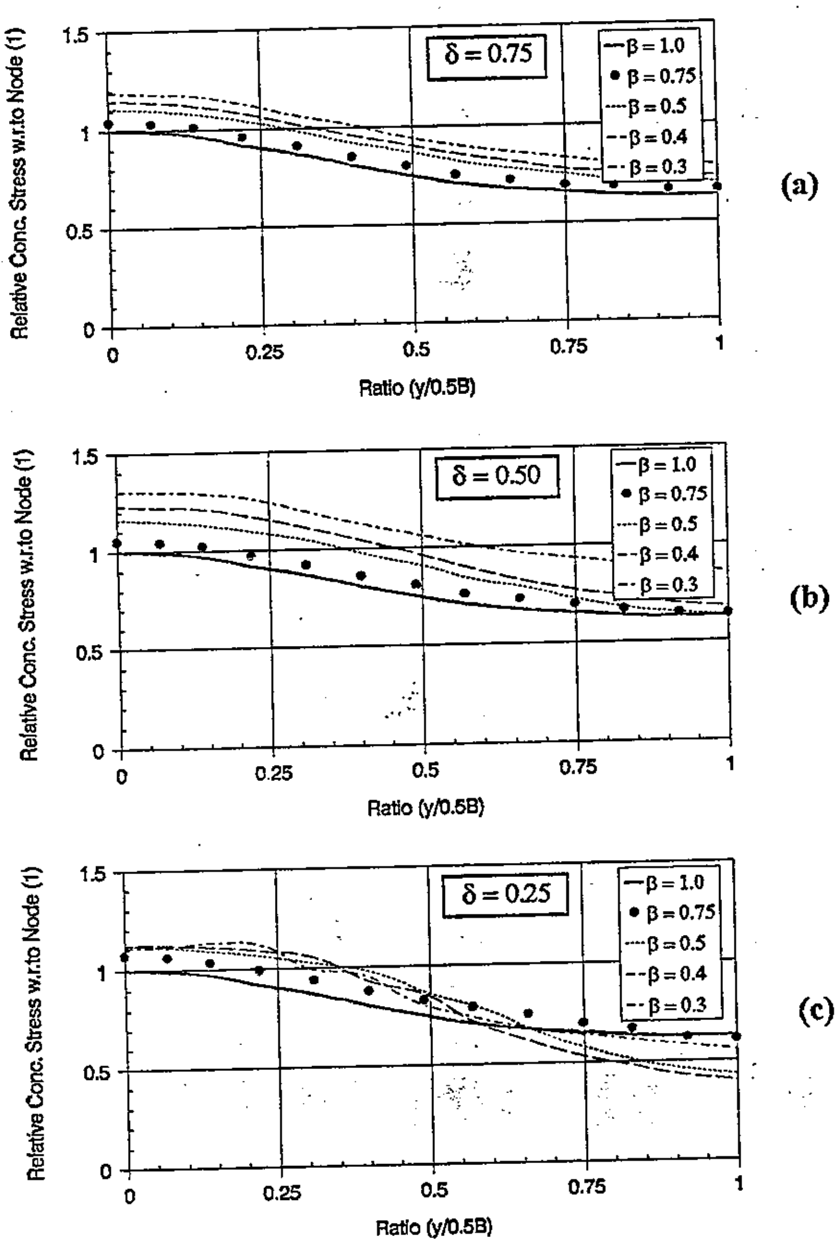


Figure (5) relative Stress in X-Direction For Parameters
$\rho = 0.50$ and $\alpha = 1.0$

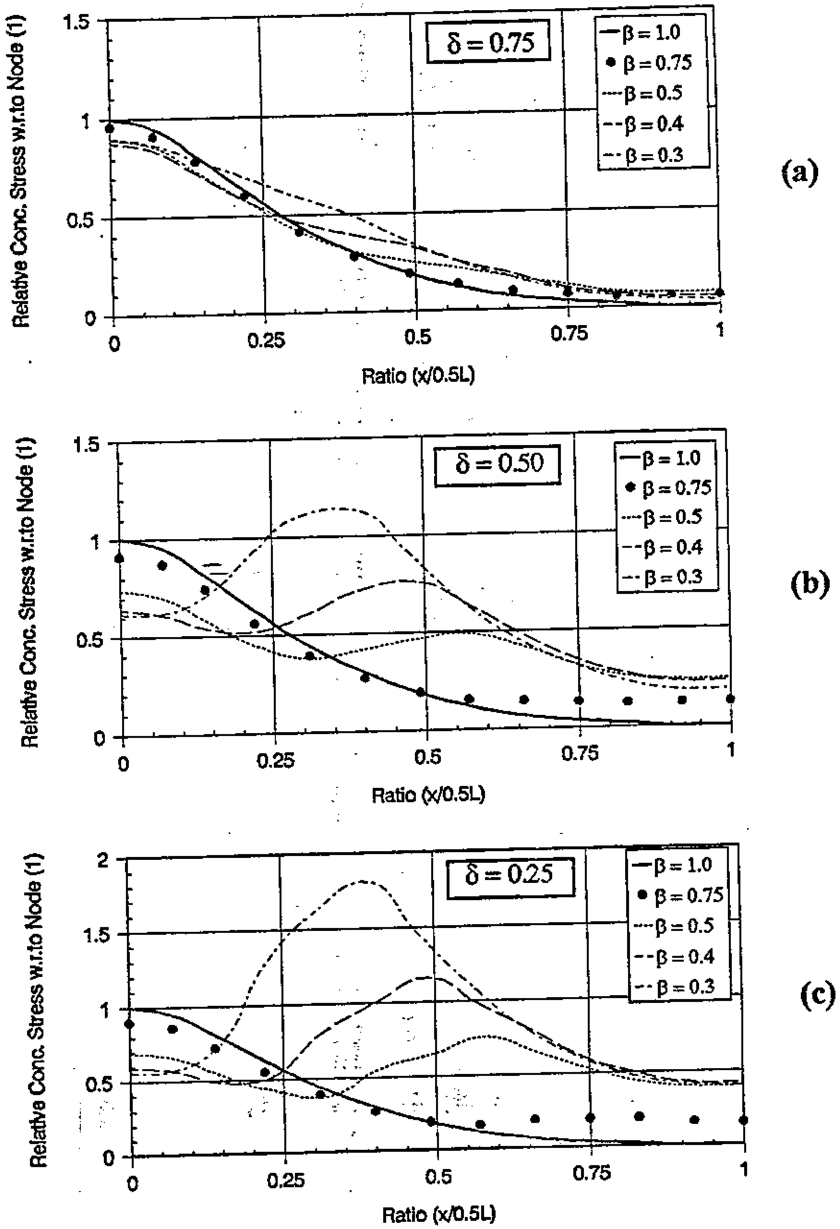


Figure (6) relative Stress in Y-Direction For Parameters
$\rho = 0.50$ and $\alpha = 1.0$

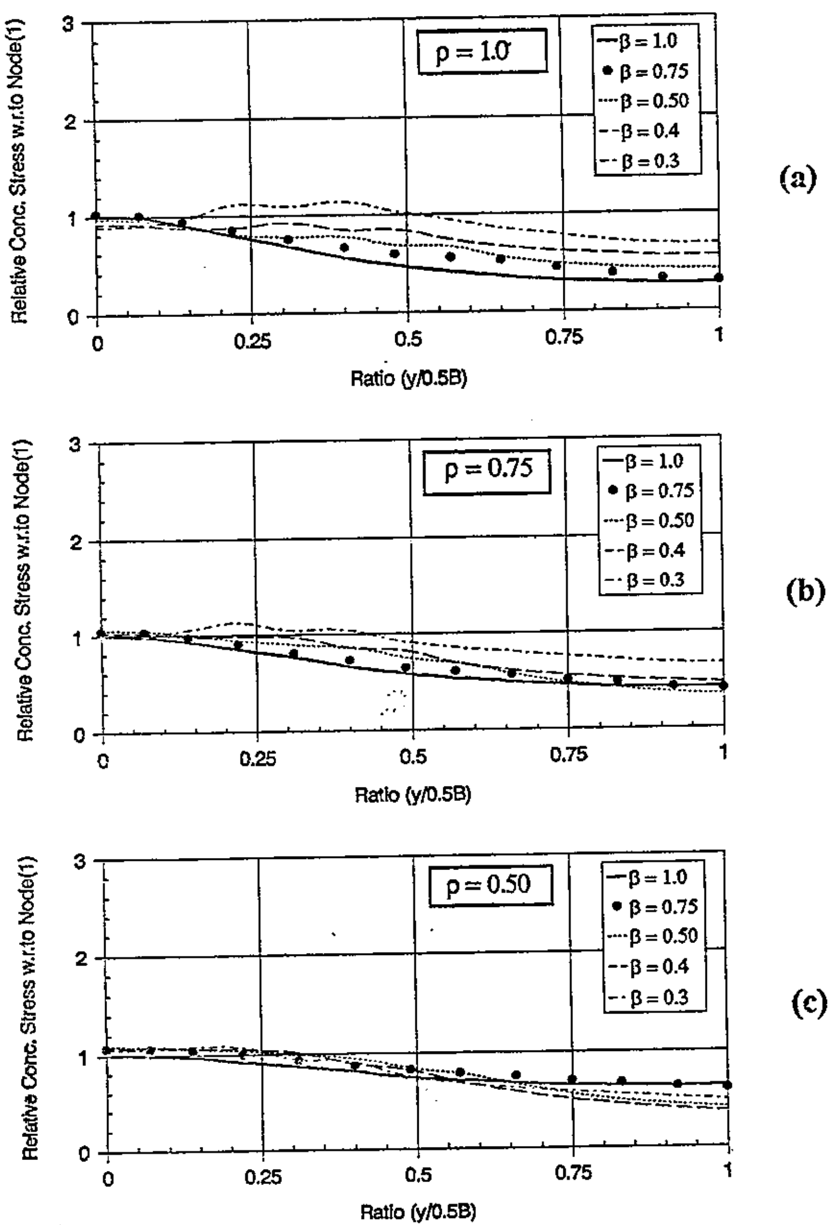


Figure (7) relative Stress in X-Direction For Parameters $\alpha = 2.0$ and $\delta = 0.25$

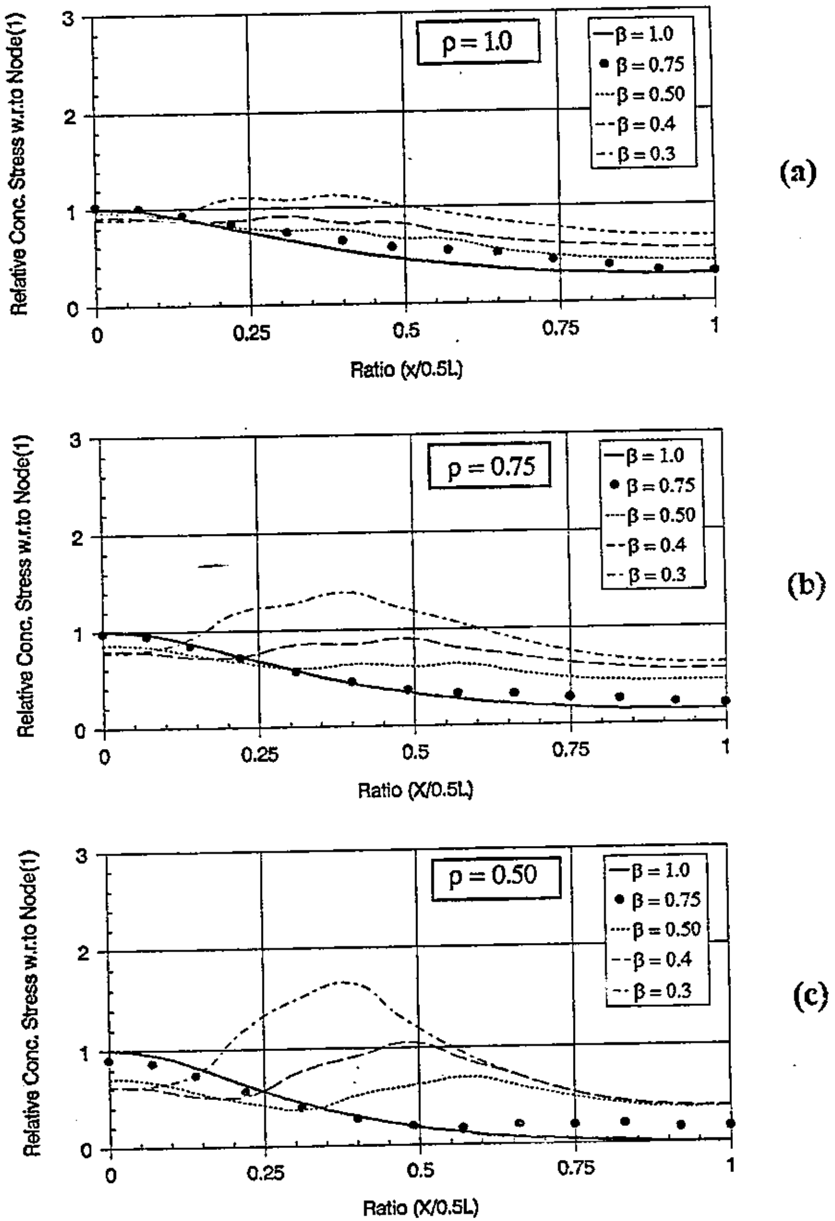


Figure (8) relative Stress in Y-Direction For Parameters $\alpha = 2.0$ and $\delta = 0.25$

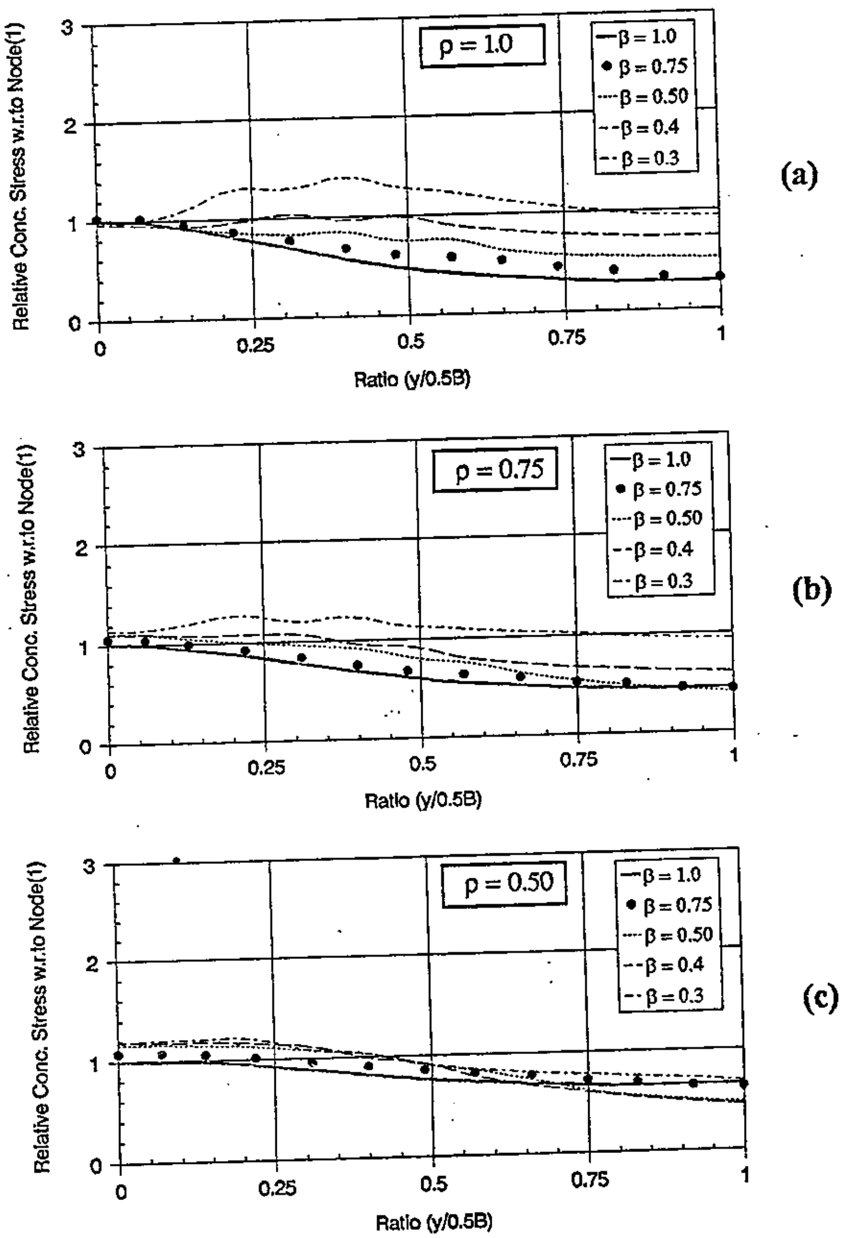


Figure (9) relative Stress in X-Direction For Parameters $\alpha = 0.50$ and $\delta = 0.25$

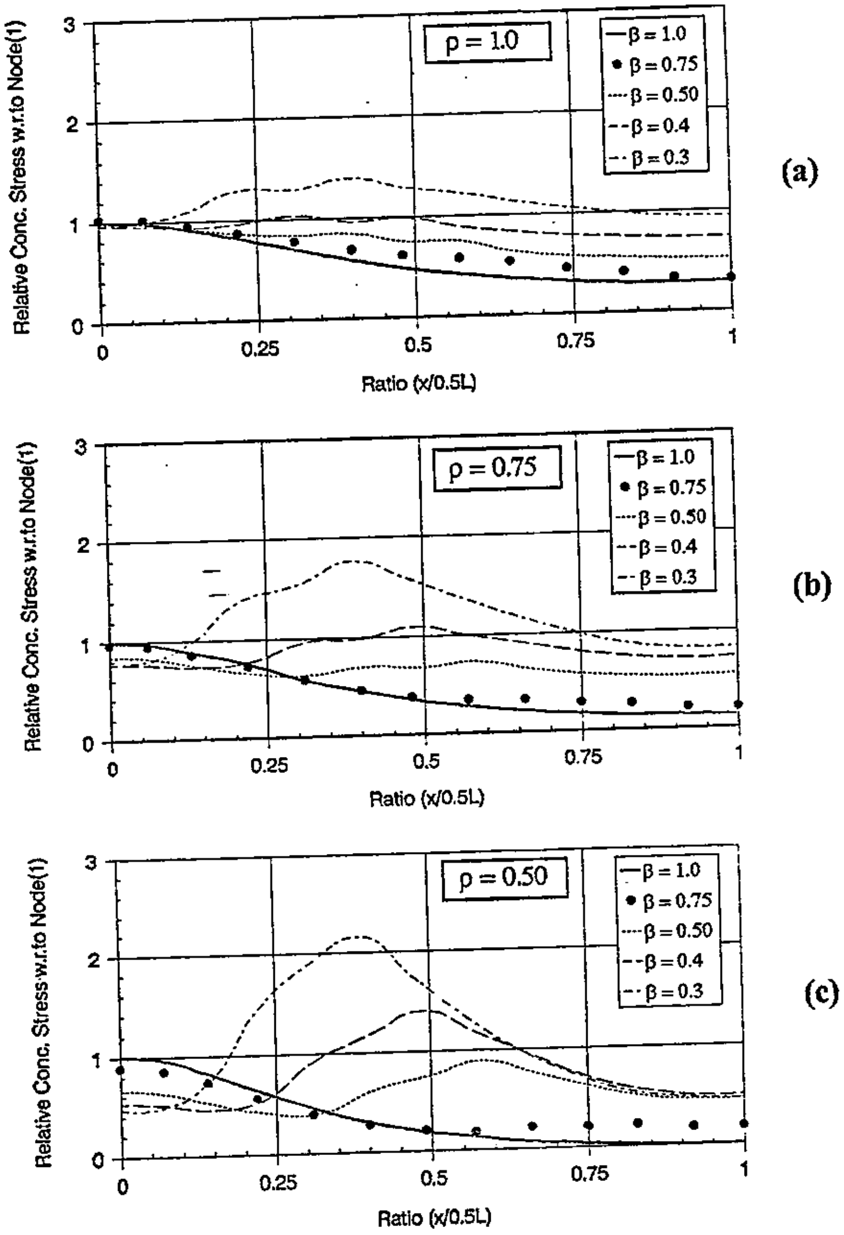


Figure (10) relative Stress in Y-Direction For Parameters $\alpha = 0.50$ and $\delta = 0.25$

Table (2) Breadth and Depth Factors and Corresponding Reduction in Concrete Volume of Foundation

Breadth factor (β)	**Depth factor (δ)**		
	$\delta = 0.75$	**$\delta = 0.50$**	**$\delta = 0.25$**
$\beta = 0.75$	10.93%	21.88%	32.81%
$\beta = 0.50$	18.75%	37.50%	56.25%
$\beta = 0.40$	21.00%	42.00%	63.00%
$\beta = 0.30$	22.75%	45.50%	68.25%

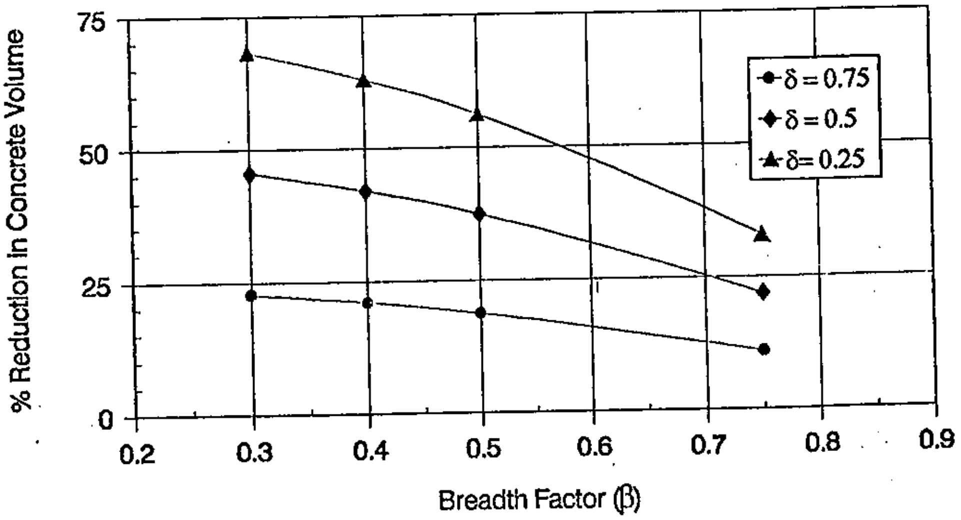


Figure (11) Breadth and Depth Factors and Corresponding Reduction in Concrete Volume of Foundation

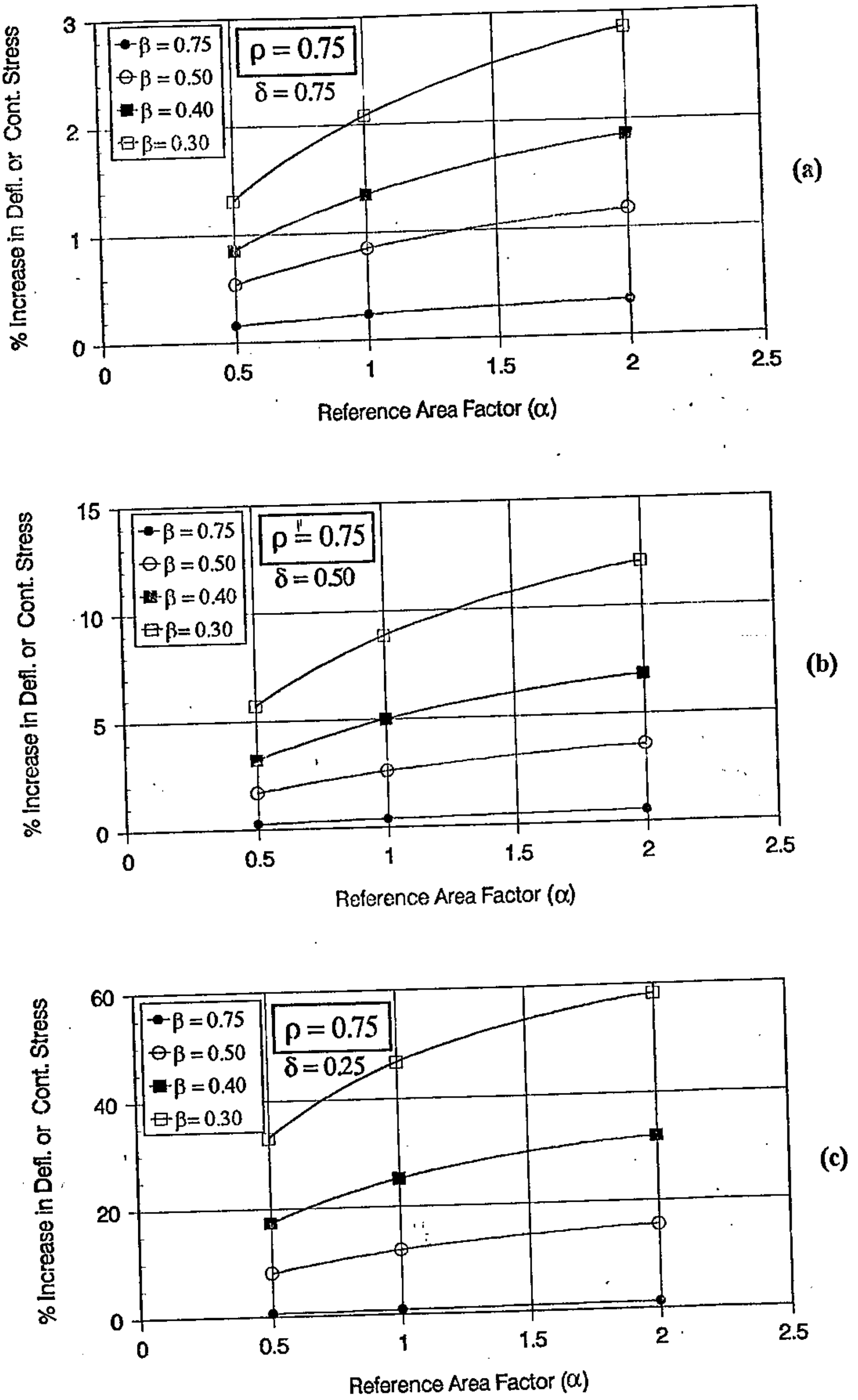


Figure (12) Relationship Between % increase in Settlement or Contact Soil Stress and Other Parameters.For Degree of Rectangularity $\rho = 0.75$

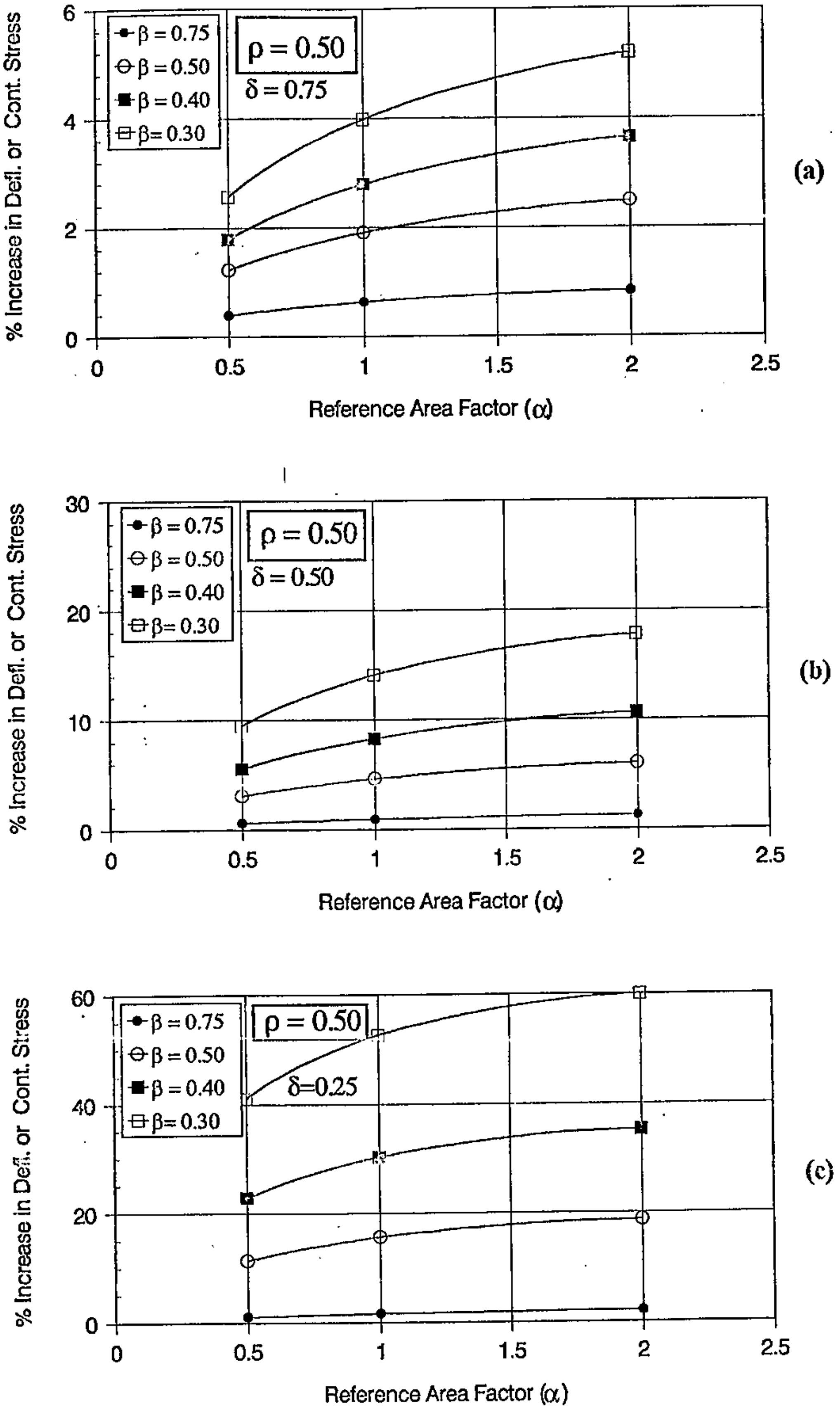


Figure (13) Relationship Between % Increase in Settlement or Contact Soil Stress and Other Parameters For Degree of Rectangularity $\rho = 0.5$