Abstract

Code Division Multiple Access (CDMA) is a widely used multiple access method in a lot of nowadays vital applications. The systems that are designed based on CDMA are suffering from multiple access interference (MAI) problem [1]. A lot of CDMA detectors are designed to overcome the (MAI) problem. But as the capability of CDMA detector in (MAI) cancellation increases, the complexity of the detector increases too [2]. This paper gives a proposal of a new linear CDMA detector that has the same multiple access interference (MAI) cancellation capability as CDMA decorrelator detector. The structure complexity of this new proposed detector is as simple as the matched filter detector structure. Solving the (MAI) problem in CDMA system with simple detector structure at the receiver helps on increasing the CDMA system capacity. The new proposed detector operation is based on the symmetry property of CDMA signatures' codes cross-correlation matrix.

Keywords

Code division multiple access (CDMA)–multiple access interference (MAI)–matched filter–decorrelator detector–minimum mean square error (MMSE) detector–signature codes correlation matrix.

1. Introduction

Linear CDMA detectors are widely used in CDMA systems' design because the complexity of these detectors is linear with the number of system’s users [2]. Matched filter, Decorrelator, and MMSE adaptive filter are examples of these linear detectors. CDMA system is interference limited system where the multiple access interference (MAI) signals from the system users that affect the desired user signal is the most influential factor on the performance of this desired user signal [3]. Matched filter detector is the simplest CDMA detector. It is the optimum receiver of a known signal in AWGN environment [1]. But in CDMA system, the matched filter is not the optimum receive because the power of system’s MAI signals is very high at the output of the matched filter [4]. So it can be said that the matched filter is the worst linear CDMA detector in the present of high system interference signals’ power. On the other hand, the decorrelator detector is the linear CDMA detector that can completely cancel all MAI signals at the output of the detector [2]. But the decorrelator detector enhances the Gaussian noise power at the detector output. Also, the structure of the decorrelator detector is quiet complex where the detector should know all the signatures’ codes of all system users in order to form the detector matrix that represents the inverse of the cross-correlation matrix among the system users’ signatures’ codes. Another source of complexity is that in decorrelator detector the received signal should first pass through a bank of K matched filters where K is the number of system users. These matched filters are matched to the signature codes of the K users. The matched filters are used to produce a vector of K-users’ energies that will be multiplied by the inverse of K×K signatures’ codes cross-correlation matrix [5]. From all of that the complexity of decorrelator detector is greater than the complexity of the matched filter detector. The MMSE detector is an adaptive algorithm detector that compromises between the matched filter detector and the decorrelator detector [6]. The MMSE detector minimizes the MAI signals’ powers and the noise power jointly at the output of the detector. The MMSE detector needs to know the desired user signature code only. So the structure of the MMSE detector is simpler than the structure of decorrelator detector. But the MMSE detector is still complex with respect to matched filter detector. MMSE detector needs a training sequence in the initiation of the communication link to adjust the MMSE adaptive filter taps. During communication course, the adaptive algorithm is working in decision directed mode to minimize the MMSE between the income signal and the detector output.

Here, a new linear CDMA detector is proposed to approach the performance of the decorrelator detector but with simpler structure as matched filter detector structure. This detector is based on a mathematical observation relating to the symmetry property of the cross-correlation matrix among the CDMA system users’ signature codes [7]. This new proposed detector with simpler structure may help in increasing CDMA system capacity by allowing more number of system’s users to share the same CDMA system’s resources.

The reminder of this paper is organized as follows. In section (2), the mathematical system model is represented. This model helped in understanding the system behavior and the problems that are faced. Section (3) shows the main idea of the new proposed detector. The system structure of the new proposed detector is also represented in this section. The probability of error in the new proposed detector is represents in section (4). This probability of error is compared with the probability of error in the case of matched filter and decorrelator detectors. In section (5), the simulation results of the proposed detector are shown. The simulations results include comparison between the proposed detector and the standard linear CDMA
detectors. These comparisons use different two criterions to have fair judgment on the new proposed detector performance. Finally the conclusions and future works are contained in section (6).

2. System Model

Multiuser CDMA detectors commonly have a front end whose objective is to obtain a discrete time process from the received continuous time waveform \( y(t) \).

\[
y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t), \quad t \in [0, T] \tag{1}
\]

The notation introduced in Eq.(1) is defined as followed.

- \( T \) is the inverse of the data rate.
- \( s_k(t) \) is the deterministic signature waveform assigned to the \( k \)th user, normalized so as to have unit energy.
  \[
  \|s_k\|^2 = \int_0^T s_k(t)^2 dt = 1 \tag{2}
  \]

The signature waveform are assumed to be zero outside the interval \([0, T]\), and therefore, there is no intersymbol interference.

- \( A_k \) is the received gain of the linear time invariant channel for user \( k \). \( A_k^2 \) is referred to as the energy of the \( k \)th user.
- \( b_k \in [-1, 1] \) is the bit transmitted by the \( k \)th user.
- \( n(t) \) is white Gaussian noise with unit power spectral density. It models thermal noise plus other noise source unrelated to the transmitted signal. According to Eq.(1) the noise power in a frequency band \( B \) is \( 2\sigma^2 B \).

Continuous to discrete time conversion can be realized by conventional sampling, or more generally, by correlation of \( y(t) \) with deterministic signals [2]. Two types of deterministic signals are of principal interest; the signature waveform and orthonormal signals [1].

One way of converting the received waveform into a discrete time process is to pass it through a bank of matched filters as shown in Fig.1. Each filter is matched to the signature waveform of a different user. In the synchronous case, the output of the bank of matched filter is shown in Eq.(3).

\[
y_1 = \int_0^T y(t) s_1(t) dt \\
y_2 = \int_0^T y(t) s_2(t) dt \\
\vdots \\
y_K = \int_0^T y(t) s_K(t) dt \tag{3}
\]

where \( y(t) \) is represent by Eq.(1). The output of the \( k \)th matched filter can be expressed as in Eq.(4).

\[
y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k \tag{4}
\]

where:

\[
\rho_{jk} = \langle s_j(t), s_k(t) \rangle = \int_0^T s_j(t) s_k(t) dt \tag{5}
\]

\[
n_k = \sigma \int_0^T n(t) s_k(t) dt \tag{6}
\]

It is noted that by Cauchy- Schwarz inequality and Eq. (2), the absolute value of the correlation coefficient is give in Eq.(7).

\[
|\rho_{jk}| = |\langle s_j(t), s_k(t) \rangle| \leq \|s_j\| \|s_k\| \tag{7}
\]

\( n_k \) is a Gaussian random variable with zero mean and variance equal to \( \sigma^2 \). It is convenient to express Eq.(4) in vector form:

\[
y = Rb + n \tag{8}
\]

where:

- \( R \) = \( \{\rho_{jk}\} = \{\langle s_j(t), s_k(t) \rangle\} \) is the normalized cross-correlation matrix.
- \( y = [y_1, y_2, \ldots, y_K]^T \)
- \( b = [b_1, b_2, \ldots, b_K]^T \)
- \( A = \text{diag}[A_1, A_2, \ldots, A_K]^T \)
- \( n \) is a zero mean Gaussian random vector with covariance matrix equal to:

\[
E[nn^T] = \sigma^2 R \tag{9}
\]

No information related to demodulation is lost by the bank of matched filters; in other words, \( y(t) \) can be replaced by “\( y \)” which is the sufficient statistic for the detection of users’ data without loss of optimality [7].

To analyze any detector whose front end consists of a bank of matched filters, the original channel model can be replaced by the linear Gaussian K-dimensional model in Eq.(8). Recall that in the synchronous model it is sufficient to restrict attention to a one shot model; thus, the dependence of \( y \), \( b \), and \( n \) on the symbol index has been omitted.

Eq.(8) can be easily generalized so as to include the complex value model in Eq.(10).

\[
y(t) = \sum_{k=1}^{K} A_k b_k s_k(t) + \sigma n(t) \tag{10}
\]
where \( y(t), A_k, s_k(t) \) and \( n(t) \) are complex values. The outputs of the matched filters are:
\[
y_k = \sum_{j \neq k} \int_0^T y(t) s_j^*(t) dt + A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k
\]  
(11)

and \( \rho_{rk} \) is defined as:
\[
\rho_{rk} = \int_0^T s_r^*(t) s_r(t) dt
\]  
(12)

Then in Eq.(13), the same vector model as in Eq.(8):
\[
y = RAb + n
\]  
(13)

can be used to represent Eq.(10) with Hermitian matrix \( R \), a complex diagonal matrix \( A \), and a complex value Gaussian vector \( n \) with independent real and imaginary components and covariance matrix equal to \( 2\sigma^2 R \).

3. Proposed CDMA Linear Detector

From the previous discussion on the linear multiuser CDMA detectors, it was found that as the capability of the detector to cancel the MAI is increased, the complexity of the detector is increased too. The simplest CDMA detector is the matched filter. But the matched filter can not cancel the multiple access interference signals as shown in Eq.(14).
\[
y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k
\]  
(14)

The detector that can cancel the MAI signals completely is the decorrelator detector. But the structure of this detector needs to know the entire signature codes of the system’s users. The decorrelator detector has a matched filter for each user signature. It calculates the correlation matrix among these users’ signature codes. Then it calculates the inverse of this correlation matrix. Finally it multiplies this correlation matrix to the outputs vector. The decorrelator detector can cancel all MAI signals but it enhances the channel noise. Eq.(15) shows the operation of the decorrelator detector [8].
\[
R^{-1}y = R^{-1}RAb = Ab
\]  
(15)

Here a new question may be answered, is it possible to have a detector that can cancel all the MAI signals with a simpler structure than the Decorrelator detector?

The two signatures decorrelator detector is the detector that may answer the previous question. The receiver structure idea is based on the symmetry property of the signatures’ codes correlation matrix [7]. The symmetry property of the correlation matrix can be represented by the following equation.
\[
\rho_{ij} = \rho_{ik} \quad \text{For all } 0 < i \& j \& k < K
\]  
(16)

where \( \rho_{ij} \) is the correlation coefficient between user \( i \) and user \( j \). Also, \( \rho_{ij} \) can be represented as the element at row \( i \) and column \( j \) in the signatures’ codes correlation matrix \( R \). The correlation matrix \( R \) can be represented as:

\[
R = S^H S = \begin{bmatrix}
    s_1^H \\
    s_2^H \\
    \vdots \\
    s_K^H
\end{bmatrix}
\]  
(17)

The operation of the two signatures decorrelator is based on using two matched filters to eliminate the MAI signals based on the symmetry property of the signatures’ codes correlation matrix. The first matched filter is the desired user matched filter that correlates the input received signal with the signature code of the desired user. The second matched filter is the reference matched filter. This matched filter is matched with a reference signature code that is not used by any user in the system. This code is common in all receivers that use the working system. Eqs.(18-19) represent the output of the desired user (user \( k \)) matched filter and the reference matched filter respectively.
\[
y_k = A_k b_k + \sum_{j \neq k} A_j b_j \rho_{jk} + n_k
\]  
(18)
\[
y_r = A_k b_k \rho_{rk} + \sum_{j \neq k} A_j b_j \rho_{jr} + n_r
\]  
(19)

where:
\[
\rho_{jk} = \int_0^T s_j(t)s_k(t) dt ; n_k = \sigma \int_0^T n(t)s_k(t) dt
\]  
(20)
\[
\rho_{jr} = \int_0^T s_j(t)s_r(t) dt ; n_r = \sigma \int_0^T n(t)s_r(t) dt
\]  
(21)

and \( s_k(t) \) is the desired user signature code and \( s_r(t) \) is the reference signature code.

From the symmetry property of the correlation signatures’ codes matrix \( R \), it was found that:
\[
\rho_{jk} = \rho_{jr} \quad \text{For all } 0 < j \& k < K
\]  
(22)

By subtracting Eq. (19) from Eq.(20), it was found that:
\[
y_k - y_r = A_k b_k (1 - \rho_{rk}) + n_k + n_r
\]  
(23)

So, the detector decision statistics can be taken to be equal to the output in Eq.(21).
\[
DS = A_k b_k (1 - \rho_{rk}) + n_k + n_r
\]  
(24)

Now it is cleared that the proposed two signatures codes decorrelator detector has canceled all the MAI signals but on the cost of duplicating the back ground channel Gaussian noise.

The detector output which represents the estimate of the desired user data will be the sign of the decision statistics as in Eq.(23).
\[
\beta_k = \text{sgn}(DS) = \text{sgn}(A_k b_k (1 - \rho_{rk}) + n_k + n_r)
\]  
(25)

Fig.2 shows the proposed two signature codes decorrelator detector structure.
The advantages of the two signatures’ codes decorrelator detector are:

1. Simple structures; the detector consists of two matched filters only instead of K matched filters as in the conventional decorrelator detector.
2. The detector does not need to know the number of system users nor the signatures’ codes of them.
3. There is no need to neither calculate the inversion of the signatures’ codes correlation matrix nor facing the problem of matrix singularity.

The disadvantages of two signatures’ codes decorrelator detector is:

1. Noise enhancing; the noise power is increased by 3dB due to the duplication of noise component in decision statistics.

As it was shown, the proposed two signatures’ codes Decorrelator detector is based on the idea of symmetry property of the correlation matrix of the CDMA signatures’ codes. But if this condition is not satisfied for certain system codes, what will be the solution in this case?

The solution of the previous problem is not difficult. By using matrix algebra, the calculation of the reference signature code will be not difficult. For any working system, the correlation vector is calculated first between the desired user signature code and the other system signatures’ codes as shown in Eq.(24).

\[ s_d = \Theta^{-1} \Delta \]

where 0 is the systems’ signature codes matrix, \( s_d \) is the desired user signature code vector, and \( \Delta \) is the correlation between the desired user code and the other users’ codes vector. The correlation vector \( \Delta \) is used to calculate the reference signature code \( s_r \) after modifying the element of index \( d \) to be equalled to \( \varepsilon \) (small number) that represents the correlation between the desired user signature code and the reference signature code. Eq. (25) shows how the reference signature code can be calculated.

\[ s_r = \Theta^{-1} \Delta \]

where \( \Delta = \begin{bmatrix} \rho_{d1} \\ \rho_{d2} \\ \vdots \\ \rho_{dk} \end{bmatrix} \]

In this case it is not necessary to have a common reference signature code for all systems receivers. On the other hand, each receiver may have its own reference signature code according to its desired user signature code as shown in Eqs.(26-25).

### 4. Probability of Error Calculations

The probability of error calculation is often depending on the detector output before the decision rule. This represents a random variable called sufficient statistics. In matched filter detector case, the probability of error of the desired user \( k \) that is a member in CDMA system is represents by Eq.(26) \[1\].

\[ P_{d,k}^{mf}(\sigma) = \frac{1}{2} \text{erfc} \left( \frac{A_k}{\sqrt{2(\sigma^2 + \sum_{j \neq k} \rho_{j,k}^2)}} \right) \]  \( \ldots \) (26)

For the case of decorrelator detector, the desired user probability of error is represented in Eq.(27) \[2\].

\[ P_{d,k}^{d}(\sigma) = \frac{1}{2} \text{erfc} \left( \frac{A_k}{\sqrt{2(\sigma^2 + \sum_{j \neq k} \rho_{j,k}^2)}} \right) \]

\[ = \frac{1}{2} \text{erfc} \left( \frac{A_k}{\sqrt{2}} \right) \]

where \( a_k \) is the \( k \)th column of \( R \) without the diagonal element, and \( R_k \) is the (K-1)x(K-1) matrix that results by striking out the \( k \)th row and column from \( R \). To obtain Eq.(27), the crosscorrelation matrix is assumed to be nonsingular.

The probability of error calculation in the case of the proposed detector is very easy. By referring to Eq.(22), the proposed detector decision statistic can be written as in Eq.(28)

\[ DS = A_k b_k (1 - \rho_{dk}) + n_k \]  \( \ldots \) (28)

where \( n_k \) is a zero mean Gaussian noise with \( 2\sigma^2 \) variance. So from Eq.(28), the proposed detector probability of error can be represented as in Eq.(29).

\[ P_{d,k}^{p}(\sigma) = \frac{1}{2} \text{erfc} \left( \frac{A_k (1 - \rho_{dk})}{\sigma} \right) \]  \( \ldots \) (29)

Fig.3 shows the plot of probability of error verses signal to noise ratio in the desired user data at signal to interference ratio of -40 dB using Eqs.(26, 27, and 29) for matched filter detector, decorrelator detector and proposed detector respectively.
5. Simulation Results

In this section, the linear CDMA multiuser detectors’ models that are presented in communication literature have been used here to compare the performance of these linear multiuser detectors simulation models with linear time invariant channel. The models that have been used are:

- Matched filter (MF) detector.
- Decorrelator detector.
- Minimum mean square error (MMSE) adaptive algorithms’ detectors such as least mean square (LMS) algorithm and recursive least squares (RLS) algorithm [9].
- Differential minimum mean square error (DMMSE) adaptive algorithm detector [10].
- Kalman adaptive filter detector [11, 12].
- Proposed two signatures codes decorrelator detector.

The performance investigation is done using two different criterions.

i. The average bit error rate criterion.

ii. The interference power measurement at the detectors’ outputs given that the signal to interference ratio at the detectors’ inputs are adjusted to two fixed different values in two different cases (-20 dB and -40 dB).

These two different criterions help in putting up a complete clear view on the performance of the linear CDMA multiuser detector simulation models that have been used in a lot of CDMA networks. Also, they will help in the comparison with the new proposed one.

The simulations are done using maximal length signature codes. The simulations are done at two different SIR values at detectors’ inputs. These SIR values are chosen to be smaller than -13 dB. From the CDG (CDMA Development Group) testing standards, the SIR value of -13 dB is the common reference value of interference at CDMA detector input in any CDMA network [13]. The average received SNR value at different detector’s inputs is varied from (-30 dB) to (10 dB).

The BER curves are plotted versus the average received signal to noise ratio at certain signal to interference ratio.

Figs.3-4 show the bit error rate curves of the pre-mentioned standard multiuser CDMA detectors using a data packet of length $10^5$ bits for 5-user CDMA system at different signal to noise ratios for linear time invariant channel. The users’ signature codes are maximal length codes of period 31. The input SIR values are -20 dB and -40 dB respectively. The average input SNR is varied between -30 dB to 10 dB steps 2 dB. The used modulation scheme was PSK for coherent modulated system and DPSK for non-coherent modulated systems.

From fig.4a, it was clear that at low signal to noise ratio, the BER of the matched filter, decorrelator and normalized MMSE detectors are better than the BER of RLS detector. On the other hand at high signal to noise ratio, the BER of the decorrelator, normalized MMSE, and RLS detectors have approximately the same BER and they are better than matched filter detector.

In fig.4b, it was shown that the BER of the proposed detector is better than the BER of Kalman filters-1&2 and DRLS detectors at low signal to noise ratio. But in high signal to noise ratio, the BER of Kalman filter-2 detector is better than BER of Kalman filter-1 detector by approximately 1 dB and it is better than the BER of the proposed detector and DRLS detector by approximately 3 dB too.

Figure 3. Probability of error for certain user in CDMA system using ML signature codes in linear time invariant channel and SIR=−40dB.

Figure 4a. Bit error rate verses average signal to noise ratio for linear CDMA multi-user standard detectors in linear time invariant channel and SIR=−20dB.

Figure 4b. Bit error rate verses average signal to noise ratio for linear CDMA multi-user standard detectors in linear time invariant channel and SIR=−20dB.
In Fig. 5a, the effect of very low signal to interference ratio (-40dB) is appeared. The BER performance of matched filter detector is roughly constant and there is no enhancement in its values with the increasing of the signal to noise ratio. However the other detectors give enhanced performance with the increasing of SNR. It is also shown that the BER performance of the Decorrelator and normalized MMSE detector is the same at low signal to noise ratio and they are better than the BER performance of RLS detector. On the other hand, at high signal to noise ratio, the BER performance of the Decorrelator and RLS detectors are approximately the same and they are better than the BER performance of the normalized MMSE detector with approximately 1.

In Fig. 5b, it was shown that the BER of the proposed detector is better than the BER of Kalman filters-1&2 and DRLS detectors at low signal to noise ratio. But in high signal to noise ratio, the BER of Kalman filter-2 detector is better than BER of Kalman filter-1 detector by approximately 1 dB and it is better than the BER of the proposed detector and DRLS detector by approximately 3 dB too.

Fig. 6-7 show the interference signals’ power at the CDMA linear detectors’ outputs. The calculations are done at the same simulation conditions as in the calculations of BER performance.

In Fig. 6, it was shown that the interfering signal power value at decorrelator and proposed detector outputs are zero however all the other detectors have a significant interference signal power values at their outputs. The normalized MMSE, RLS, Kalman filter-2 and DRLS detectors have approximately the same interfering signal power value at their outputs. Kalman filter-1 detector output has an interfering signal power value greater than the interfering signal power value of the other detectors except the matched filter detector which has the highest interfering signal power value at its output.

In Fig. 6, it was shown that the interfering signal power value at decorrelator and proposed detector outputs are zero however all the other detectors have a significant interference signal power values at their outputs. The normalized MMSE, RLS, Kalman filter-2 and DRLS detectors have approximately the same interfering signal power value at their outputs. Kalman filter-1 detector output has an interfering signal power value greater than the interfering signal power value of the other detectors except the matched filter detector which has the highest interfering signal power value at its output.

In Fig. 7, it was shown that the interfering signal power value at Decorrelator and proposed detector outputs are also zero however all the other detectors have a approximately the same interference signal power values at their outputs except the matched filter detector which has the highest interfering signal power value at its output.

Fig. 6-7 show the interference signals’ power at the CDMA linear detectors’ outputs. The calculations are done at the same simulation conditions as in the calculations of BER performance.
6. Conclusions and Future Work

The Decorrelator detector needs to know all the users’ signatures codes in CDMA system. Also, this type of detectors needs to calculate the correlation matrix among these users’ codes and the inversion of this matrix. These requirements may not be easy to achieve. The new proposed detector that is based on two matched filters only does not need to know all signature codes of all system users nor calculation of correlation matrix and its inversion. This new proposed detector is as simple as the matched filter detector but has the same MAI cancellation of the decorrelator detector. The disadvantage of this new proposed detector is the duplication of system noise.

This new proposed linear detector is based on a mathematical observation relating to the symmetry property of the cross-correlation matrix among the CDMA system users’ signature codes. This new proposed detector with simpler structure may help in increasing CDMA system capacity by allowing more number of system’s users to share the same CDMA system’s resources.

In the near future, it may be required to have a numerical expression for the bit error rate of the proposed detector as the expressions that are stated in communication literatures for the standard linear CDMA detectors. Also, it may be interested to have an evaluation of the new proposed detector performance in time varying channels such as Rayleigh and Nakagami fading channels.

7. References